UNIVERSITY OF TORONTO
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ELASTICITY AND STRENGTH
OF
MATERIALS USED IN ENGINEERING CONSTRUCTION

SECTION IV

COLUMNS
Elastic Theory and Economic Design
AND
Analysis of Eye Bars
Pin and Rivet Connections

BY

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ADVANCE NOTICE

SECTION V will deal with the economic theory of bridge construction, truss types, economic triangulation, economic floor system and paneling and comparison of the various types and their fitness or adaptability to different locations.

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PREFACE

Whether as engineers we are exercising our thinking faculties to develop structural practice within the straight and narrow path of correct theory, or are straying therefrom step by step by imitating the work of others can be best disclosed by the historical review.

Thus a study of the development of the form of heavy column sections in the past discloses the saving by many of our bridge engineers of thirty cents worth of pin plate by concentrating the metal in the web at the expense of the flange at a sacrifice of a dollar's worth of strength in the main member. The underlying cause of the Quebec collapse was the adherence to this bad practice, the last step only was the dangerous adoption of a seemingly conservative increase in the slenderness ratios over that of the Blackwells Island Bridge chord and those of other bridges.

The development of the Rankine Eulerian theory of columns has been again and again presented in works on mechanics of materials without change for a generation after the practical builder had determined by experience that end conditions in practice were unrelated to the coefficients of the theory. Notwithstanding a half century of experience with struts of hollow and unsymmetrical makeup our writers on mechanics still present the old formulas for a solid strut applied without suggested modification to the diverse forms of fabricated sections in common use.

The identity of strength of hollow tubes with spherical ends and hinged ends is noted and the non-identity of strength of angle and T struts with spherical bearings and hinged bearings are investigated and the reason for the divergence in strength is found in the different twisting resistances of the different disposition of the metal.

During the displacement of iron by mild steel pioneers in its use proclaimed its superior qualities, greater homogeneity and strength than that of wrought iron. In substantiation of these claims the ability of the softer grades of steel to stand excessive drifting, bending and doubling of the metal upon itself without reference to the resistance or work required to cause such distortion developed misleading standards for determination of the relative fitness of the material for the work it is to perform in the structural frame.

The lack of precise knowledge regarding the relative resistance to compression of wrought iron and steel is clearly illustrated by an endeavor in the preliminary report of the special Committee of the American Society on columns to reduce test results upon wrought iron sections to their equivalent in steel by the addition of twenty percent to the iron values when the old wrought iron columns fabricated from metal having a tensile strength of 50,000 pounds per square inch developed under compression considerably greater strength than steel columns with a metal of 60,000 pounds tensile strength. A discussion of the quality of metal from the economic standpoint involving a bird's eye view of the possibilities of further development is presented in connection with the discussion of economic design of compression members.

Stress line analysis as developed in Section II of this series is applied to disclose the condition of stress in the solid column and in demonstrating the manner in which the material obeys Hooke's law. Heretofore column action had been regarded much as described by Horace E. Horton, Trans. Am. Soc. C. E. LXXII, p. 461—

> "It is subtle and illusive, and may side-step"
> "It is composite, and not homogeneous".
> "No law is written as to the proper relation and proportion of its constituent parts."

In developing a rational theory to bridge this void in exact knowledge of this important subject, the writer has touched upon the errors of assumption which underlie the complex theory of secondary stress which is the bane of the practical designer today and applying the criterion of historical review traces the development of these cumbersome theories to the failure of badly designed German bridges in which the collapse may be more rationally accounted for by the lack of understanding of the proper design of the main compression members independent of the secondary stresses due to displacement of the action line of the stress in the plane of the truss. Stress line analysis is applied to the eye-bar head and the distribution of stress through the bar itself disclosing the reason for the greater elongation of the metal toward the end than in the center and an approximation to the proper proportion of the head if the bar is to break in the body and the distortion of the head is not to be excessive. The departure in large sized eye bars from those proportions found to test satisfactorily in the smaller bars has been touched upon and the substantiality of the criticism seems warranted by the usual failure in the heads of a number of the large bars tested for the information of the Board of Engineers of the Quebec Bridge.

The rising working values in shear and bearing for rivets has been traced historically and the elastic theory applied to the determination of the magnitude of the principal elastic stresses to which the material is subjected before it commences to receive a permanent set. The result of such analysis indicates that the resulting principal stresses so greatly exceeds the apparent shear stress figured on the cross sectional area of the rivet that a return to the more conservative values used for shear and bearing in the days of wrought iron and the early introduction of steel is in order.

In the early design of the pin connected bridge truss it was thought necessary to figure the pins for shear only and not for moment. Why the correction and rectification of this deficient practice was not extended to the rivet which as the parts commence to slide is subjected to bending, axial tension and shear, is an oversight in theoretical treatment that it is difficult to account for. When the designers of our great structures are conscientiously endeavoring to keep working values within the elastic limits of the material and are going to such extremes as that of providing for secondary stresses in the main members which exist more predominently in the imaginative assumption of the computer than in the finished structure, consistency demands that the humble rivet, though of insignificant size receive the same careful attention for upon it the stability and endurance of our structure depends. Upon what theory do we consistently keep the working stresses in the body of the member well below the yield point value of the metal and increase them in the bearing on the pin plate approximating the yield point of the metal? Figuratively, the inconsistency of such practice comes within the Levitic category of straining at a gnat and swallowing a camel without salt.

May, 1924. C. A. P. TURNER.

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CHAPTER XVII

AXIAL STRESS AND BENDING COMBINED

1. Introdutory: Every horizontal member, subject to longitudinal compression or tension, presents a state of combined axial stress and bending. Application of the longitudinal stress eccentrically gives rise to a like state of stress. If the member is short and the lateral deflection small, simple superposition of axial stress upon the state of bending stress caused by the eccentricity of application of the axial force is sufficient for practical purposes; whereas, if the member is long, its treatment becomes more complex.

2. Strength of a Hook or Block Under Eccentric Loading: Fig. *A* represents a hook to which the axial pull P is applied at an eccentric distance x from the center of gravity of mid section.

Fig. *B* represents a short block subject to a compressive force P, a distance x from the center of gravity of its cross section.

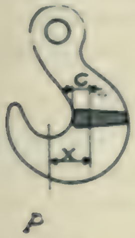

Fig. A.

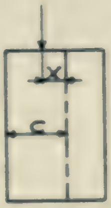

Fig. B.

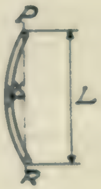

Fig. C.

For each case the axial stress equals

$$f_a = P/A \qquad (1)$$

The bending stress equals

$$f_b = \frac{(Px)c}{I} = \frac{(Px)c}{Ar^2} \qquad (2)$$

in which r is the radius of gyration of the section. Therefore,

$$f_a + f_b = f = P/A(1 + cx/r^2) \qquad (3)$$

equals the maximum fiber stress, tension for the hook, or compression for the block.

3. Rankine's Formula for columns up to forty diameters may be derived from equation (*3*) as follows: Suppose the load P to be applied to the column ends axially on knife edge bearings and to be of sufficient magnitude that a certain sidewise bending takes place measured by a central ordinate x. As in the case of the block, the bending moment from which fb is derived equals Px. By the theory of flexure the deflection of the locus of pressure equals the square of the length L multiplied by a coefficient representing the elastic properties of the material and the end conditions divided by C the distance of the outer fiber to the neutral plane. Substituting this value of x in (*3*), we may write:

$$f = P/A\left(1 + a\frac{L^2}{r^2}\right) \text{.........} (4)$$

which is Rankine's formula for columns of ordinary length which fail by bending and crushing on the concave side of the member. The evaluation of the coefficient (a) for different conditions will be investigated in connection with Euler's formula in the following article.

4. Euler's Formula for the ultimate strength of very long columns is derived on the assumption that failure is caused solely by the fiber stress of flexure; i.e. that the component compression P/A may be neglected in comparison with the far greater fiber stress of bending. As before, the moment of flexure equals:

$$M = Px \text{.........} (5)$$

Treating the moment areas as substantially parabolic for practical purposes, the lateral deflection equals

$$x = \frac{{}^{*}5}{48}\,\frac{ML^2}{EI} \text{.........} (6)$$

Substituting this value of x in (5)

$$P = 9.6\frac{EI}{L^2} = \text{appr.}\ \frac{\pi^2 EI}{L^2} \text{.........} (7)$$

Which latter is Euler's formula for the ultimate strength of a very long column hinged at its end.

From (7) it is obvious that the magnitude of P is independent of the magnitude of the lateral deflection x. Elastic resistance to lateral bending increases with an increase of deflection but the lever of the bending moment which causesd eflection increases in like ratio since the deflection under the pure bending assumed is the lever of the applied forces. The state of equilibrum under the load P determined by the formula is unstable within the elastic limits of the material of the strut or spring and represents its ultimate resistance.

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*5/48 is slightly less than the exact coefficient of the sign curve of moment area.

When the column is cylindrical this equation becomes

$$P=\pi^3Ed^4/64L^2=Ed^4/2L^2 \text{ app.}$$

And for a square column

$$P=\pi^2Ed^4/12L^2=5Ed^4/6L^2 \text{ app.}$$

Church in his Applied Mechanics gives the following interesting application of Euler's formula to a very thin steel ruler: $L=30$ inches; $D=1/30$th inch; width 2 inches; $P=3\times(10)^8\div162{,}000\times900=2.03$ pounds.

Experiment showed that a small addition to a load of two pounds caused a large increase of sidewise bending.

A clear idea of the deportment of the long column and the law of increase of its fiber stress with increase of load may be obtained by the following simple experiment with a 42″ T-square with a blade of wood $2\frac{1}{2}'' \times 1/8''$. Such a square approximates in length the height of a roll top desk. Stand the blade vertically parallel to the face of the end of the desk; secure a number of half inch square steel bars 18″ long as weights. Place the ends of two bars one on each outer end of the head and incline them toward each other resting the other end on a book on the desk top. Then weight the blade by placing one at a time bars crossing the first pair but near to the head. Compute the reaction on the head from the position of the weights and measure the deflection of the blade outward from the face of the desk.

Up to two pounds no *perceptible deflection appears; then with an increase of weight a small deflection is noted which increases with each additional weight added while the lateral stability of the bent blade decreases so that with each added weight a greater and greater deflection in proportion to the weight occurs until the weight reaches the limit of the resiliance of the blade (i.e. P of Euler's

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*The student's attention, in connection with this phenomena, is referred to Art. 319 Rankine, Applied Mechanics.

Theorem. A spring of a given length and section, to the ends of whose neutral surface a pair of forces are applied will not bend if the forces are less than a certain finite magnitude.

formula) when equilibrium becomes unstable and the blade will continue to bend until it dumps its load or fails in the process.

5. The End Condition of the Column, whether free to turn or clamped and fixed, is a large factor in the determination of the intensity of the working stress and ultimate strength.

Case I: (Solid Sections) Both Ends Clamped: Embodies a constraint of elastic bending expressed by the condition that $dx/dy=0$ for $y=0$ and $y=l$ in which as before x is the horizontal ordinate of deflection and y the vertical coordinate of any point along the length of the column.

The curvature (exaggerated) will be that represented by Fig. D analogous to the curvature of a beam restrained at both ends under UNIFORM loading. Sections thru the points of inflection *i-i* by virtue of the end restraint are subject to thrust without bending, hence the column length between the points of inflection *i-i* presents a condition identically the same as a column with hinged ends treated in Arts. 3 and 4, with the length decreased as the reciprocal of the square root of three.

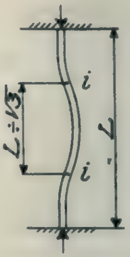


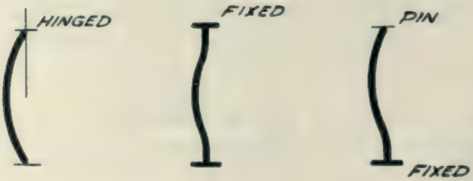
Fig. D.

Inserting this value in (4), Rankine's formula for the column with solid symmetrical section with fixed ends becomes

$$f=P/A(1+aL^2/3r^2) \quad \ldots\ldots\ldots\ldots (4)a$$

And Euler's formula for ultimate strength for columns with fixed ends likewise becomes

$$P=3\pi^2EI/L^2 \quad \ldots\ldots\ldots\ldots (7a)$$



END CONDITIONS

Case II: One End Fixed and the other End Hinged is intermediate between two ends fixed and two ends hinged so that the numerical factor *n* to be inserted in the Rankine and Euler formula is 2 instead of 3 as in Case I.

Case III: Flat Ends is the term designating a square milled end bearing which fixes the column end up to a slenderness ratio of about 100 L/r for a symmetrical section when the bending of the section commences to relieve the pressure on the outer portion of the bearing area toward the convex side and the deportment of the strut approaches more and more closely as the length increases to that of the column with hinged ends, but the ultimate strength remains greater even for such large ratios of L/r as four hundred because of the favorable shift of the center of application of the load with the bending of the shaft toward the corner thus reducing its lever arm.

Case IV: Round Ends: The round end, such as a ball and socket bearing for a symmetrical solid shaft, will develop substantially the same strength as a hinged end which has the load applied to a knife edge bearing at the center of each end. When the section is unsymmetrical such as a *T*, a pair of angles, a single angle or a Z-column section with 4 *Zs* and a plate, the hinged bearing will develop greater strength than the ball and socket bearing because the hinge largely increases the torsional rigidity over the ball and socket bearing and the increase in ultimate strength developed by the hinged end over that of the round end represents the change in torsional couple affecting the ultimate resistance. The marked degree of twisting of Z-sections tested by the Bureau of Standards is a matter that is today too well known to require further comment.

The preceding investigation differs from that customarily found in text books on mechanics of materials. Church assumes that because the tangent to the elastic curve of the column is zero at the point of restraint as far as bending is concerned that the end quarters of the column are in the same condition as the center quarters reversed but as was shown in Section II the form of the deflection curve is determined by the double summation of moment and slope areas.

In the derivation of the formula the ordinate along the length of the column being *x*, varies as a curvilinear function with the bending of the column and not as a linear function which is the sole condition under which the points of inflection locate themselves at the quarter points.

6. Stress Line Analysis of column action may assist greatly in clarifying the nature of the resistance presented. If we conceive of an ideal concentric application of the load upon parallel milled column ends on frictionless surfaces then the principal compressions lie parallel to the axis of the column, the principal tensions lie normal to the axis and the curves of greatest shear would make an angle of forty five degrees to the axis. The manner in which this stress distribution is modified in service may be investigated for any given non-uniform distribution of compression over the end sections by the principles applied in Section II to beams.

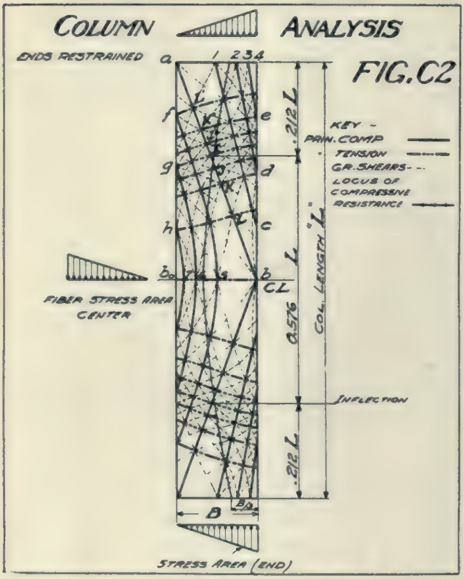


FIG. C2

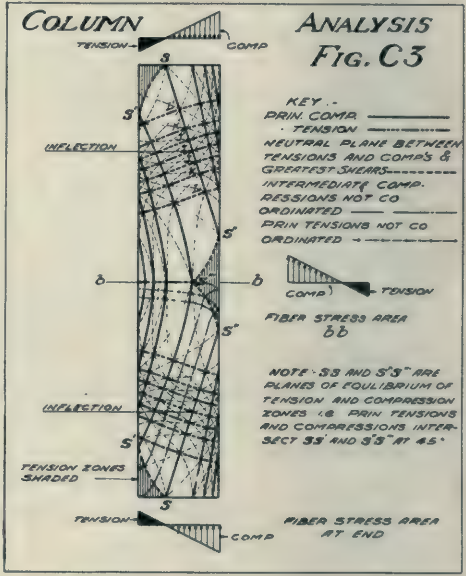


FIG. C3
COLUMN ANALYSIS
TENSION
COMP
S
S'
S"
S'
S
INFLECTION
INFLECTION
b
b
KEY .-
PRIN. COMP.
TENSION
NEUTRAL PLANE BETWEEN TENSIONS AND COMP'S & GREATEST SHEARS
INTERMEDIATE COMPRESSIONS NOT CO ORDINATED
PRIN TENSIONS NOT CO ORDINATED
COMP
TENSION
FIBER STRESS AREA bb
NOTE - SS AND S'S" ARE PLANES OF EQULIBRIUM OF TENSION AND COMPRESSION ZONES I.E. PRIN TENSIONS AND COMPRESSIONS INTERSECT SS' AND S'S" AT 45°
TENSION ZONES SHADED
TENSION
COMP
FIBER STRESS AREA AT END

In Fig. C2, assume full restraint at the column ends and a fiber stress variation from zero at the left to a maximum at the right side of the shaft. Since action and reaction must be equal and opposite the *same fiber stress area reversed must lie at *b.b* the mid section of the columns. Divide the fiber stress area at the bearing into four equal divisions at 1—2—3 and 4, *B*/2—.7071*B*, and .866*B* respectively from *a*.

Since the restraint imposes no condition governing the direction of the principal compressions at the column end we may conclude

*Rigidly true before appreciable bending occurs.

that they maintain the straight line character of the previous case, i.e., that they incline toward the axis, as *ab*,—1*c*,—2*d*,—3*e*, etc.

As the intensity of the stress along *ab* is zero at the ends (by virture of the stress areas assumed it will represent the curve of equilibrium of the shearing stresses which adjust or govern the distribution of the longitudinal compressions upon consecutive cross sections. As the fiber stress area follows the lineal law in its variation the shear stress intensity along *ab* would follow the parabolic law and the coordinated divisions *J′*, *K′*, *L′*—*J*, *K*, *L* are determined as in the case of the beams Section II. As the compression at the boundary varies from zero at the center to a maximum at 4 the points *c d e* at the extremities of 1*c*—2*d*—3*e* will divide *b*4 into equal parts by Hooke's law. Unlike the principal compressions 1*c*,—2*d*,—3*e*, the principal compressions *f*5—*g*6—*h*7 must meet the conditions of restraint at *b*,*b* and become parallel to the axis at 5, 6 and 7 since a principal stress cannot change abruptly in direction in an elastic media. And moreover each of these curves must intersect the principal tension b_0b at right angles. These curves may be treated without material error as parabolic and the principal tensions be drawn thru *I*, *J*, *K*, *L*, *J′*, *K′*, *L′*, normal to the principal compressions. The curves of greatest shear will be the diagonals of the trapezodial figures formed by the coordinated curves of principal tension and compression.

From the divergent conditions governing and controlling the direction of the principal compressions in the end and central quarter lengths of the column it is obvious that the points of inflection cannot locate themselves at the quarter points as commonly though erroneously assumed.

Stress analysis C3 considers part of the stress area tension instead of all compression from it we observe that as the stress changes from all compression across the end faces and central section to part tension and part compression the locus of equilibration of the longitudinal shears moves toward the *Column Ends;* thereby accounting in part for the very rapid rise in the value of the controlling maximum fiber stress after this change has occurred.

Before the outer fiber on one side is subjected to tension the bending moment is measured by a couple whose lever arm for fixed ends is $d/3$ or less than $d/3$, and the magnitude of the force forming this couple equals the stress volume of the triangular area whose base is the diameter and altitude the difference between the maximum and minimum outer fiber compression at the center of length. Since the moment of a couple is the same about any point in a plane the principle of least work requires that resistance of the unbalanced compression center about the extreme edge. The moment of inertia is therefore four times as great as in simple flexure. The variation in the moments of inertia and section modulii for different degrees of change from all compression longitudinally of the shaft to tension on one side and compression on the other, may be readily investigated by the formula for the value of I' for a shift of the axis from that of the center of gravity of the section $I' = I_0 + Ah^2$.

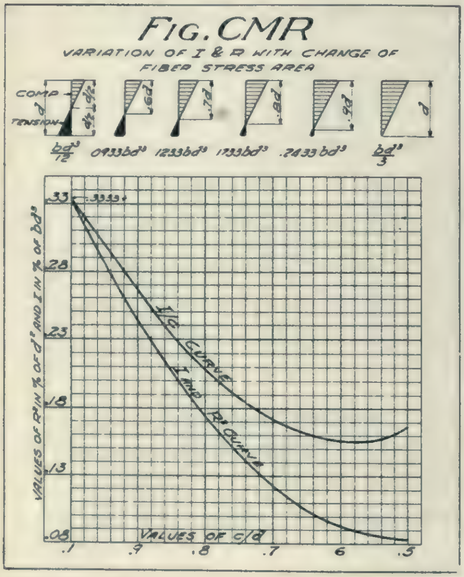


FIG. CMR

VARIATION OF I & R WITH CHANGE OF FIBER STRESS AREA

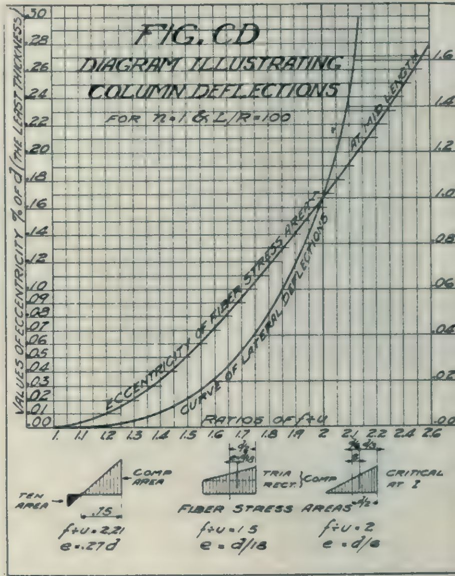


FIG. CD

DIAGRAM ILLUSTRATING COLUMN DEFLECTIONS

FOR n=1 & L/R=100

Fig. CMR gives a plot of these values of I in decimals of bd^3 and of R^2 in decimals of d^2 and also a curve of the values of I'/C in which C is the distance of the extreme fiber in compression from the axis or center of rotation. When the total longitudinal stress upon normal sections is compressive the fiber stress area of compression at mid length of the column is trapezoidal made up of a rectangular component giving rise to uniform longitudinal compression (i.e. without bending) and a triangular compressive stress area giving rise to both shortening and bending. The state of stress produced by this triangular component area is that represented for fixed ends in Fig. C2, in which the principal tension on the normal mid section is at right angles to the axis and the principal compressions are parallel to the axis of the shaft.

Bending is thus induced by the equilibration of unbalanced shearing distortion instead of being substantially measured by the magnitude of the major plus and minus fiber elongations in parallel as in beam action.

The relation of the major stresses of compression in parallel and tension at right angles to the axis of the shaft instead of both in parallel with it at mid length is so antipodal to the state of stress in beam action that rigid reasoning as to column bending by analogy to the bending of beams in simple flexure leads to erroneous results.

Thus the bending of a beam is about the neutral axis which at mid span is at the center of the symmetrical section and is the plane of equilibration of the longitudinal shears. In the column problem of Fig. C2 the equilibration of the longitudinal shears is unrelated to the neutral plane of beam action intersecting the outer edge at mid length and end. The moment of inertia and square of the radius of gyration opposing the bending couple is therefore four times as great as though the resistance were analogous to beam action and the corresponding deflection resulting from a given eccentricity of loading is one fourth as great as by analogy to beam action and the increase in fiber stress but one eighth as great.

When the state of stress changes to tension on one side the moment of inertia undergoes that rapid change shown in Fig. C.M.R. and small increments of load then cause such large increments of lateral deflection that for the practical column in bridge and building work this point of change may be considered a critical one approximating closely the useful strength of the member when L/R is less than 120.

The rapid reduction in lateral rigidity after this critical change in the fiber stress area occurs accounts for the curious deportment of the T-square blade in the experiments described in (4).

7. History and Use of Rankine's Formula: Its form was first proposed by Tredgold on theoretical grounds, using in place of L/r for the slenderness ratio, L/d in which d was taken as the out to out of the solid or hollow cylindrical cast iron section. Hodgkinson's empirical formula replaced it for a time when it was revived by Mr. Lewis Gordon who evaluated constants for the formula from the experiments of Hodgkinson. The formula in that form was known as Tredgold-Gordon formula but because it did not take into consideration the different distributions of the metal and make up of the section where it was not solid, Rankine proposed the substitution of L/r as the slenderness ratio in place of L/d and the formula thus modified became known as the Rankine formula or Rankine-Gordon formula.

In figuring the ultimate strength, Gordon took for wrought iron $f=36{,}000$; for cast iron 80,000; and the coefficient $a=1/3000$ for wrought iron; for a cast iron hollow cylinder 1/800 and for cast iron solid cylinder 1/400. The column with fixed ends was taken as being four times instead of three times as strong as the column with round ends.

In this respect the formula disagreed with Hodgkinson's experiments in which the long solid prismatic column was found to be only three times as strong with fixed ends as with round ends as we have shown should be the case upon theoretical grounds.

The ultimate strength so figured was customarily divided by a factor of four in design. In later practice building wrought iron bridges, see Vol. VIII, Trans. Am. Soc. C. E., the following modification of Rankine's formula was used to a large extent as shown in the following formulas in which

p = the mean compressive strength on the cross section

L = the length in inches

r = the radius of gyration of the section, in inches.

$$p = \frac{8000}{1+L^2/40000\,r^2} \text{ for square end compression members}$$

$$p = \frac{8000}{1+L^2/30000\,r^2} \text{ for compression members with one pin and one square end}$$

$$p = \frac{8000}{1+L^2/20000\,r^2} \text{ for compression members with pin bearings.}$$

While in 1898 Mr. C. C. Schneider used for soft and medium steel the following modification of the same formula:

$$p = \frac{15{,}000}{1+L^2/13{,}500\,r^2} \text{ for soft steel}$$

$$p = \frac{17{,}000}{1+L^2/11{,}000\,r^2} \text{ for medium steel.}$$

It is to be noted that in the early design, of wrought iron bridges in this country that the erroneous coefficient n was not used, as the difference between a pin end and a fixed end was covered by a coefficient equaling the reciprocal of 40,000 for fixed ends and the reciprocal of 30,000 for pin and square ends and the reciprocal of 20,000 for two pin ends. Further, experiment on pin ends brought to light the fact that nearly the same strength was developed for the pin end as for the square or fixed and where the length was not too great.

8. Rational Column Formulas: All column formulas in use with the exception of Euler's are empirical, i.e., derived from the result of tests to rupture of the member. Euler's formula, because it disregards direct compression is applicable to only very long columns such as are rarely used in engineering structures. Because of the injurious effect of range of stress when exceeding the natural limits of elasticity of the material it is very desirable to develop column theory in such wise that the magnitude of the elastic stress under the applied load will be known.

To derive such a formula, let f equal the maximum compressive stress on the concave side of the column, let u equal P/A the average compression over the cross section of the piece. Following the general reasoning of Tredgold and Rankine, this leads to the expression

$$u = \frac{f}{1 \times q\dfrac{L^2}{r^2}} \qquad \text{(1)}$$

in which q varies as the numbers 1, 1/2, 1/3, etc., for different end conditions.

This formula for the medium length column is to be combined with that of Euler for long columns because the gradation from the short to the long column must be gradual and not abrupt. The reasoning of Euler may be expressed by the equation.

$$u = \frac{\pi^2 E}{n}\ \frac{r^2}{L^2}. \qquad \text{(2)}$$

in which E is the coefficient of elasticity and n has the values above noted.

Rational column theory must embrace both of these cases. For the very long column it should reduce to substantially Euler's expression. For a short block or prism where L is relatively small with respect to r it should reduce to $u=f$ or the unit load equals the unit stress.

Let L/r be regarded as the abscissas of a curve and be denoted by x. Let the unit load u be the ordinate and be denoted by y. Then the formulas of Rankine and of Euler may be written

$$y=\frac{f}{1+q\,x^2} \text{ and } y=\frac{\pi^2E}{nx^2}$$

Now in order that these two formulas may be combined into one, the curves represented by these equations must be tangent to each other, and this condition will determine the value of q. For the point of tangency the two ordinates are equal, and also the two derivatives of y with respect to x. The two derivatives are

$$\frac{dy}{dx}=-\frac{2\,f\,q\,x}{(1\times q\,x^2)^2} \text{ and } \frac{dy}{dx}=-\frac{2\pi^2Ex}{n\,x^4}$$

Placing equal to each other the two values of y and also the two values of $\frac{dy}{dx}$, and solving for x and q, there is found,

$$x_1=\infty \text{ and } q=\frac{nf}{\pi^2E}$$

The point of tangency, therefore, is when L is infinite with respect to x, and for this point the ordinate $y^1=0$.

The value of q being now determined, its insertion in Rankine's expression gives the formula

$$*u=\frac{f}{1+\frac{nf}{\pi^2E}\,\frac{L^2}{r^2}} \quad \text{.............................} (3)$$

---

*Eq. (3) was developed by Crehore in 1879 but unfortunately he adopted Euler's erroneous treatment of end conditions and his work therefore failed of fruition in the field of practical design.

Johnson's modern framed structure has platted a curve derived from Crehore's formula by dividing the yield point value **f** by four after reducing it by (3) an equally useless form.

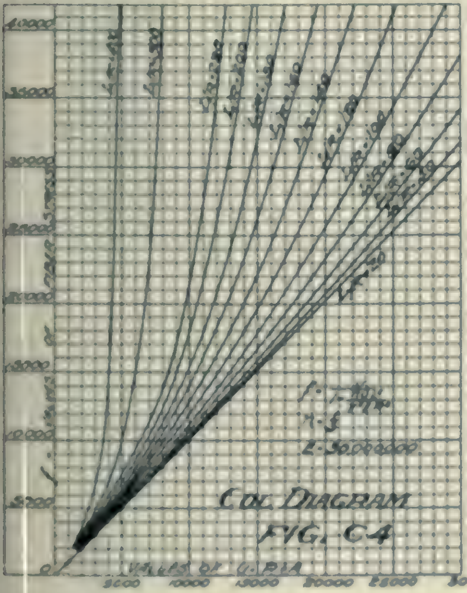
Solving for f

$$f = \frac{u}{1 - \frac{nu}{\pi^2 E} \frac{L^2}{r^2}}$$

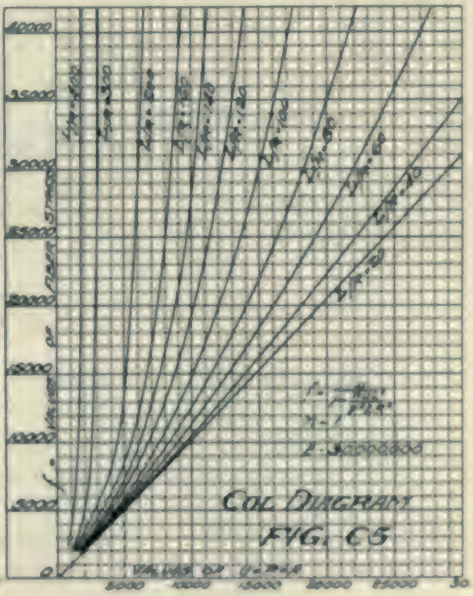
The formula shows that the length can be several times larger than r before the greatest compression f exceeds the average working stress u by an appreciable amount. For example, in a case of a soft steel column with round ends loaded so that $u = 10{,}000$ pounds per square inch for $L/r = 1$ the unit stress f equals 10,000.4 pounds. For L/r equals 7, $f = 10{,}020$ pounds or for $L/r = 10$, f equals 10,040 pounds.

In other words where the length is but four to five times the thickness the stress is substantially pure compression. When L/R is 300, the strength computed by (3) is ten percent less than by Euler's formula and for $L/R = 500$ it is four percent less, agreeing with Euler's values when $L = \infty$.

9. Diagrams for Solid Columns: The practical application of the rational formula may be simplified by diagrams such as Fig. C4 for solid prismatic columns having fixed ends. Plot the values of



COL DIAGRAM
FIG. C4



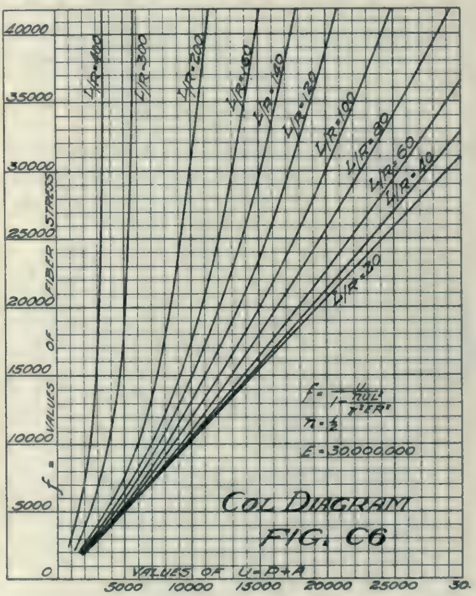
COL DIAGRAM
FIG. C5

f as ordinates and of u as abscisas. Compute sufficient values to plot the different curves for different values of L/R. Then for any given value of L/R the value of u for a fixed fiber stress f may be read at once from the diagram.

Fig. C5 is a similar diagram for solid prismatic columns having round ends, figured for $E=30{,}000{,}000$ suitable for medium or intermediate grade of steel.

For timber the mean value of E may be taken at 1,500,000; cast iron 15,000,000; wrought iron 25,000,000; soft steel 28,000,000; medium or intermediate grade 30,000,000.

Fig. C6 is a similar diagram computed for $n=1/2$, a close approximate to the strength of a column having one round and one fixed end or one having pin ends in which the diameter of the pin is not less than one third the width of the member.



COL. DIAGRAM
FIG. C6

10. Safe Working Stresses in any framed structure should be so selected that the compression and tension members possess a like capacity to withstand over load. In tension the maximum fiber stress increases substantially in proportion to the load applied but in the compression member as appears from our rational formula the maximum fiber stress increases in more rapid ratio than the increase of the mean or average unit stress.

If (for structural grade steel) 18,000 lbs. per square inch be selected as the working stress in tension (when the static stresses are increased by an impact allowance) then the working stress to match for compression should be half the mean compression "u" which will produce according to the formula a maximum fiber stress of twice 18,000 pounds per square inch, thus providing for the same overload capacity for the compression as for the tension member. For different grades of steel similar proceedure should be followed:

11. Column Deflections: The amount which a column under load bends or deflects has no direct proportionality to the amount of the load, nor, in fact is it proportional to the apparent eccentricity of the fiber stress area at mid length, nor to the ratio of the mean compressive stress (u) to the maximum (f). However, when f/u equals unity (e) the eccentricity equals zero and the deflection also is zero.

When f/u at mid length equals 2, the fiber stress area is triangular and entirely compressive (i.e.) zero on the convex side and increasing according to the linear law to a maximum on the concave side. Relative deflections for a given ratio of L/R may be plotted to illustrate typical column deportment treating the deflection when $f/u = 2$ as unity as follows:

*In Fig. C.D. as absissas plot the values of f/u from unity at the left increasing toward the right. From (1) as the origin plot the ordinates of eccentricity so that at $x=2$, $y=.16\frac{2}{3}=1/6$ at some convenient scale. Through 2 as a new origin draw a vertical representing the scale of relative deflections such that at $x=2$ and $y=0.16\frac{2}{3}$ the deflection equals unity. Then as the deflections for values of $f/u=2$ are of a much smaller order than the eccentricity of the fiber stress area, the relative deflections within this limit will vary substantially as the products of the eccentricities multiplied by the triangular component of the fiber stress area or D the relative

$$\text{deflection } = e \times u \frac{(f-u)}{u} = e(f-u)$$

In evaluating it is to be noted that for each different value of e, u differs in relative magnitude as shown by our rational formula.

The curve of relative deflections shows that the solid column of uniform material obeys Hooke's law in its elastic lateral deflections since these deflections are proportional to the product of two factors

*Fig. CD, P. 8.

(i.e.) the eccentricity at mid length of the total fiber stress area representing lever arm and the magnitude of the triangular component of that area represents the force giving rise to the bending through its lever arm e.

For the same section and given ratio of f/u the triangular stress area or force varies with the ratio of L/R according to our rational theory. This reasonable variation has heretofore seemed so unaccountable that theoretical writers have erroneously endeavored to explain it on the ground of accidental flaws in the metal, residual strain from manufacture, or wide variation of initial eccentricity instead of investigating the sink holes of assumption underlying the extensive muskeg of algebraic equations by which the elastic action of the column has been obscured.

The lateral deflection of the column is a continuous function. The fact that the deflection is almost negligible at the critical load under which the fiber stress area changes from total compression to part tension and reduces to but a minute fraction of this almost negligible amount under working loads has given rise to the erroneous supposition that the deflection was a discontinuous function absolutely instead of approximately zero under ordinary working loads.

In test specimens except those in which the slenderness ratio L/R is large, no tension parallel to the axis will occur in any of the fibers until the column may be said to have failed. The common idea of regarding the bending of a column is erroneous, founded on the impression obtained from the curve of flexure used in derivation of the formula indicating a considerably bent member, with a tension and compression side as in a beam. This idea is strengthened by the appearance of photographs of specimens which have failed under compression representing the conditions long after the maximum load has passed and the specimen has greatly deflected under reduced load thus giving a false impression of the true column action.

Associated in the formation of these incorrect impressions is the idea that the equilibrium of the fiber stress areas taken place along the axis or center line of the shaft instead of along a curved line as indicated in our Fig. C2. Examination of this figure and the curved gravity axis of the fiber stress areas indicates immediately that the lever arm of the load producing the bending may be far greater than the deflection or bending of the column.

Remarks: The inadmissability of the assumtpion that the states of stress in the initial stages of flexure of a column are comparable to beam action is here investigated for the first time and the error of Euler's comparison of pin and fixed ended columns disclosed. The rational formula for the maximum fiber stress under given working loads may be experimentally verified and the derivation for solid sections of safe working stresses therefrom substantiated.

Safe working stresses for the compression members in the balanced design of a frame should possess the same over load capacity that the tension members provide for as treated in the last article. The modification of the rational formulas for application to built up and hollow sections will be dealt with in the following chapter.

In developing the rational formula for fiber stress *I* and *R* are the minimum for the cross sectional area, so fixed by the predominent influence of bending in the long column.

The reader will find a complete bibliography of column investigation and prior literature in Salmon's work on Columns published by the Oxford Press.

CHAPTER XVIII

ECONOMIC THEORY OF STEEL COLUMN DESIGN

1. Introductory: Because the safe unit stress decreases materially as the slenderness ratio L/R increases various unsymmetrical rolled sections such as rolled angles and tees or double angles are customarily used in roof trusses and rolled H-sections, built zee-bar columns, plate and channel sections, Phoenix, etc., as illustrated in Fig. C6 are employed for columns in building work.

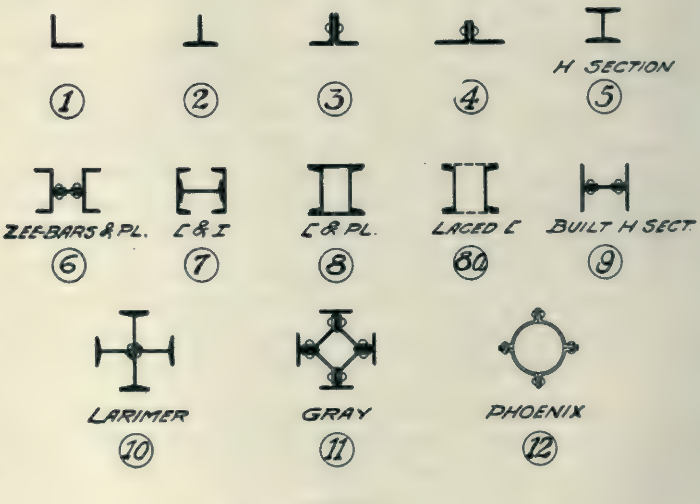


Fig. C6.

TYPICAL COLUMN SECTIONS IN BUILDING CONSTRUCTION.

TYPICAL BRIDGE COMPRESSION MEMBERS

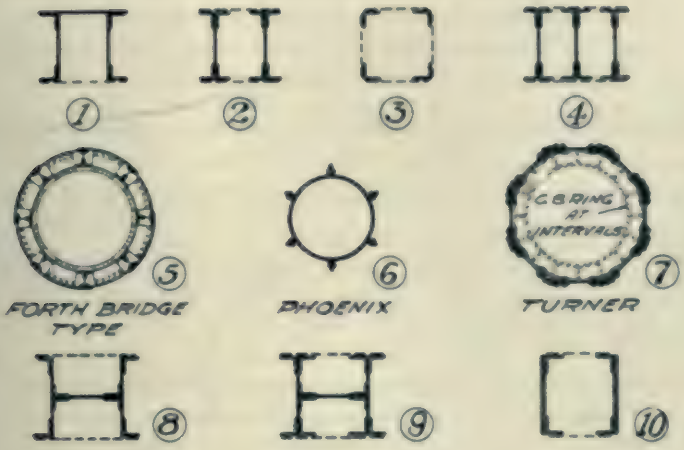


Fig. C7.

Plate C7 illustrates typical sections used in Bridge construction. If an attempt be made to analyze these sections by a strip theory their general divergence from the solid is strikingly apparent.

If sections (1) to (10) of Fig. C6 be divided into longitudinal strips each presents a slenderness ratio of a far higher order than that of the section as a whole, and hence the total resistance becomes some function of the particular combination of strips presented by the configuration of the section, requiring investigation of the various typical forms to determine how the formulas for the solid section should be modified for application to their design.

A strip at the extremity of an angle, tee, H or Zee-bar section is relatively lacking in lateral rigidity and would tend to buckle out of line normal to the direction which the strut tends to bend under axial compression.

Such tendency toward lateral buckling gives rise to the twisting action presented by the Zee-column and other sections when tested to destruction. (See Bureau of Standards Tests).

Accordingly, the extent of this torsional stress in the unsymmetrical section is to be investigated in addition to the column action typical of the solid symmetrical section.

2. Twist: Twist about the column axis under axial compression cannot occur in a symmetrical solid shaft such as a round or square because such deformation would increase the length of the shaft (see Section III, Chapter XIII) and that under a force tending to shorten it is contrary to Newton's third law.

Therefore, the solid shaft cannot twist out from under its load and its deformation is pure crushing and bending and the same reasoning applies to the hollow cylinder and sections 5-6-7 of Fig. 7.

In the UNSYMMETRICAL section the phenomena of twist occurs and for a given twisting force (in this case the lateral buckling tendency of the strips) the angular distortion increases as the free length of the shaft twisted.

For round ends this deformation may run the full length of the shaft but if the ends be pin connected or clamped they are fixed against torsional rotation at the ends and the free length is diminished to $L/4$ and the torsional component in like ratio.

In tests of angle and T-struts by Mr. Christie, Mem. Am. Soc. C. E. round ended struts (ball and socket bearing) developed an ultimate strength per square inch less than solid struts of the same slenderness ratio in terms of L/R. Moreover, the ultimate strength of the round end was considerably less than the same strut pin ended with greased pins and for the very long strut but about one fourth (instead of one third that of fixed ended struts of solid section.)

These tests indicate that the tendency to buckle of the extremity of a strip having a flange projecting about six times its thickness weakens the strut at the extreme fiber an amount equal to that of column action of bending for a round ended strut under axial application of the load and for the pin ended strut an amount one fourth as great as the stress reduction for column bending. Accordingly, for the solid column of a given ratio of L/R where f' is the maximum fiber stress and $u'=P/a$ then the increased difference between f and u for the built section should be $2(f'—u')$ for round ends and $5/4$ $(f'—u')$ for pin and square ends. For different ratios by the theory of flexure these deductions increase inversely as the square of the ratio of the thickness to the projecting length.

3. The Buckling of the Web Plates in Built Channels of a laced column is to be figured in a similar manner. The ratio of the stiffness of a projecting cantilever to that of the plate restrained at its edges would be about as one to twenty five. The ratio of the thickness to unsupported length being six for the projecting flange, the corresponding thickness of the web between points of restraint should approximate thirty for the same reduction from the values permitted for column bending in planes normal to the web. For smaller unsupported thicknesses the rule of increased deduction inversely as the cube of the thickness divided by the unsupported distance applies.

Because the stiffness of the member as a whole when we consider the buckling of the web and the crippling of the flange is influenced by the bending resistance of the lacing in the side planes, angle iron lacing is to be preferred to bar lacing because of its greater rigidity under both twisting and bending forces. In figuring the thirty thicknesses of the plate this should be taken between rivet centers where the built channel is made up of two angles and a plate and from the edge of the angles where four angles and a web instead of two angles and a web comprise the sections laced together.

4. **Secondary Column Action** occurs in the section of the built or rolled channel between the points of attachment of the lattice bars. As these points of attachment are not rigidly fixed this reduction may be figured with practical precision by applying the general formula to the sub-section between lattice points, taking $n = 1/2$. The radius of gyration should be enough greater in the plane of the lattice to compensate for this secondary effect.

5. **The Maximum Intensity of Stress in Latticing** is related to the make up and proportions of the sections laced together. If the slenderness ratio L/R of the sections laced together is such that without lacing the separate built channels could carry the larger part of the load then the stress upon the lattice becomes relatively small and its section should be proportioned with this fact in view. On the other hand, if the slenderness ratio of the built channel exceeds 250 then the cross section of the lacing should not underrun the proportionate areas found satisfactory in practice for laced rolled channels, to wit:

For 15″ channels 2½″ bars × 7/16″ min.*
For 12″ channels 2½″ bars × 7/16″ min.*
For 10″ channels 2¼″ bars × 3/8″ min.*
For 8″ channels 2″ bars × 5/16″ min.*
For 6″ channels 2″ bars × 1/4″ min.*

---

*Thickness to be not less than $L/40$ single and $L/60$ double lacing. L = distance between rivets.

The proportionate cross section of lacing may logically be figured from the relative minimum area required for the heavy channel reduced in keeping with the smaller slenderness ratio of the built sections or parts laced together. If the net section of the lacing bar for a 15—33# channel is three quarters of a sq. in. then the net section of the bar is 7.5% that of the channel section. Now if we have two built channels or I sections of plates and four angles having each a cross section of 100 square inches the angle lacing may be less than seven and a half square inches in section inversely as the value of L/R of the section between the apices of the angle lacing is less than that of the channel between the attachments of the bar lattice. Thus for a 40″ × 1¼″ web and four angles 6 × 6 × 1″ webs 40″ centers, 4 × 3½ × 7/16 angles at 60° to the axis would represent proportionate strength if each angle be connected by four 1″ round rivets. Single lacing at 60 degrees to the axis weighs less than double lattice at 45 degrees and while the latter gives perhaps an appearance of greater strength it does not seem substantiated by experiment.

In the experiments at Watertown Arsenal 1909-10, 8″ × 13.75# channels 5½″ back to back with 2 × 3/8″ bars 6″ alternate spacing developed the same strength as double lattice of 2 × 5/16″ bars 8½″ centers. With single lacing there is less pantograph action tending to bend the channel at rigid batten ties or stays, hence the better showing of single lacing in proportion to the weight of metal. The pantograph action accompanying the distortion of lacing indicates that for sections such as (8) and (9) of Fig. C7 batten plates or flat channel stays are preferable to lacing for staying and stiffening the outer flanges in the plane shown dotted in section. Whereas, lacing or lattice is suitable for types 1-2—3-4 and 10 of Fig. C7. Double lattice, supplemented by transverse ties is objectionable since the spreading or pantograph action just discussed is restrained by the tie and the diagonal lattice is thereby subjected to half the unit compression of the shaft reducing its capacity as bracing in like measure.

Stress analysis shows the greatest shear strain in the solid section at the point of inflection of the solid shaft with fixed ends and close to the end of the column with pin ends and by analogy the maximum lattice strain should occur toward the end of the column with pin ends and in the vicinity of the fifth point of the length with restrained ends.

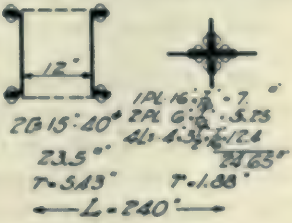
6. The Relative Area of Flange and Web in built channel sections from the standpoint of efficiency is indicated by general experience with solid rolled sections. With the rolled structural channel the web area of standard minimum sections is substantially half the total area or less and the breadth of flange is one fourth to one third the depth.

Because in theory we treat the laminated built channel as though solid, accrediting to it a degree of rigidity more or less imperfectly developed by the riveting it is bad practice to add materially to this imperfection by increase of the web at the expense of flange width and area which determines the rigidity of the section without proper reduction of the working stress, a percentage equivalent to half the percent of the unfavorable change in proportion.

7. Multiple Web Sections such as (4) of PL C7 are sometimes used to maintain the above favorable ratios of web to flange. With the three webs the unsupported length of the lattice is reduced but the two outside webs are called upon by virtue of their position with but two thirds the metal to resist the buckling tendency of the member about an axis parallel to the web. Hence the radius of gyration should be fifty percent greater about this axis than about the normal axis due allowance being made for end conditions.

8. Deficient Riveting of Thick Laminated Webs may in general be disclosed by stress analysis following the method of Fig. C2, wherever the efficient ratios of web and flange areas have been disregarded. This deficiency under secondary column action between lacing frequently reduces the strength half or third thereby accounting for the low effective elastic limit of many tests of scale models of large bridge members and their low ultimate strength compared to that of solid laced channel sections of the same material and like slenderness ratios.

9. Computation. To illustrate the application of the foregoing principles compute sections (a) and (b) for ultimate resistance the ends being fixed.



(*a*) (*b*)

Yield point = 35,000 (soft steel)

For (*a*) $L/R = 44.2$; $u = 31,000$, diagram C4.

Lacing 18″ centers of connection f'-u' = 4000.

Deduct for buckling stress 1000.

Deduct for secondary lacing 1000.

Elastic strength 29,000 × 23.5 = 685 000#.

For (*b*) $L/R = 127$ $f' = 21{,}000$. $t/y = 9$ for projecting pl.
$(f'\text{-}u') = 14{,}000$. $t/y = 6$ for thickness of L^s & pl.
$14{,}000 \times .25 \times (3/2)^2 = 7{,}200$ lbs.
$21{,}000 - 7{,}200 = 13{,}800$.
Elastic strength $= 24.65 \times 13{,}800 = 342{,}000$.#
Or the elastic strength for (b) is three-eighths that of (a).

Were the steel of intermediate grade instead of soft, i.e., 75,000# ultimate with a yield point of 43,000 in place of soft steel 52,000 to 58,000# per square inch ultimate strength with a yield point of 35,000# per square inch, the resistance of each section would be increased one sixth approximately.

The computation indicates the economic advantage of favorable arrangement of the material. In (b) the thin projecting plates add but little to the strength of the section beyond the angle flanges and the efficiency of a quarter of the section is reduced to an inconsiderable amount.

Again, comparing the elastic strength thus figured with that given by the old formula for the ultimate strength $\dfrac{45{,}000}{1+36{,}000\ L^2/R}$ we note that the computed ultimate strength of *b* is approximately 32,500 pounds per square inch against 15,750# above figured. It appears from this that the dependable strength of the column of built channels is but ninety percent of that resistance credited to it by old formulas whereas the badly proportioned star and tee sections may be no more than fifty percent as strong.

As the strength of a triangular beam simply supported with an apex downward may be increased in strength by planing off the apex so may such a column section as (b) be increased in elastic strength nearly 20% by cutting down the unsupported projecting plates which weaken it by the twist of their buckling tendency.

10. Column Weakness vs. Secondary Stress: In the preceding discussion it has been shown that the strength of a non-solid section may be far less than that of a solid or symmetrical hollow section. The relative weakness of a star section of two plates and four angles appears from the computation in the last section and for like reason the ultimate strength of a tee section having a deep web and two angle flange such as a thirty by a half web and two angles 5× 3½ × ½ for a relative slenderness ratio of 120, compared to a section of two channels laced is low because of the tendency of the web or flanges to twist out of line. Twisting weakness of such star and tee shaped compression members readily accounts for the failure of certain German bridges without resort to the complex theory of secondary stress evolved by Manderla, Breslau-Muller, Ritter to explain their collapse. As two unrelated causes cannot account in toto for a given phenomena a consideration of the assumptions on which the common theory of secondary stresses is founded may be in order.

(a) That the secondary effects are confined to bending of the member in the plane of the truss.

(b) That the gusset connections at the joints may be assumed rigid, while the members connected by the joint plates are treated as elastic.

(c) That the moment of inertia controlling the flexure of the strut under a given displacement of the center of pressure is no greater than in a case of simple flexure.

Assumption (a) is inadmissable for torsion is by far the greatest and most dangerous of indirect stresses: (b) is inconsistent with elastic theory and (c) is four fold in error.

The tedious computation by the theories mentioned under these unwarranted assumptions give computed secondary fiber stresses so many times greater than actually occur under pressure from the pure bending in the plane of the truss that such computation may in general be dismissed as valueless. Proper attention should be given to the make up of the section in the design of compression members and the torsion resulting from the primary compression stress whenever the section is unsymmetrical about the center of section.

11. The Bending Moment the Column is Subjected to by Its Load cannot be figured as the product of the applied force times an ordinate represented by the sum of eccentricity of the applied force with respect to the axis of the resisting material plus the deflection of the column axis for the reason that the flexure of the column at its critical state of stress is small, almost negligible. But we know that the column has been approaching the critical point of failure without material deflection. Hence we must infer that the moment has been increasing largely without corresponding bending, notwithstanding that the load may have been applied on knife edge bearings to avoid eccentric application.

The resistance of the column along its length is centered on the locus of consecutive gravity centers of transverse fiber stress areas normal to the plane of bending or weakest direction. The locus of these centers of gravity of the fiber stress areas has been determined for a load producing a critical stress in the stress line analysis of the column with fixed ends, Fig. C2. This curve is of the same species regardless of the end conditions just the same as the curve of moments is determined by the loads and their point of application on the beam regardless of whether the beam be fixed at its ends or simply supported. No reversal in curvature of the locus of gravity centers of the fiber stress areas is occasioned by clamping or fixing the column ends.

The curve of consecutive centers of gravity of the compressive fiber stress areas may be otherwise regarded as the LOCUS OF COMPRESSIVE REACTION to the load the column or strut is called upon to carry. The product of the load times the ordinate of the curve from a line joining its extremities measures the moment of the external forces in a manner comparable to both simply supported and restrained beams and as in the case of these different beams the end conditions determine or regulate the distribution of the internal resisting moments. But this locus of compression differs from the moment diagram of the beam in that the ratio of its ordinates to the intensity of the external forces is not a fixed ratio as in beam action. For example, with hinged ends and $L/R=100$ under half the ultimate load the middle ordinate is approximately $d/60$ approaching $d/6$ as the ultimate strength of the strut is approached when measured from the axis of the strut as a base.

12. Experimental Verification of the Theory of Twisting and Bending is to be found by comparing the tests of symmetrical sections with those of unsymmetrical sections with round, hinged and fixed or clamped ends.

Experiments at the Watertown Arsenal in 1909-10 on welded tubes with spherical ends and welded tubes with hinged or pin ends, show no difference in average strength between the spherical ended and pin ended specimens. This series has been plotted by Salmon in his work on columns, page 219 in a manner disclosing this relation at a glance.

For unsymmetrical sections we may refer to the elaborate series of experiments of Mr. Christie in 1883 described in the Transactions, Am. Soc. C. E., April 1884. The ends of the struts were arranged, first with the ends fitted to hemispherical balls and sockets, second with the ends hinged on cylindrical pins, and third with the ends rigidly clamped. The load was weighed with a Fairbanks testing machine and increased until failure occurred.

The measured deflection of the struts in Mr. Christie's experiments substantiate the curve of lateral deflections of Fig. C. D. of the previous chapter when consideration is given to the fact that some of the *Ts* and angles were cold straightened and the deportment of the specimen is occasionally erratic for that reason. Mr. Christie's experiments on wrought iron welded tubes show that for the ball and socket end connections the ultimate strength developed for large ratios of *L/R* are nearly twice that for *Ts* and angles with similar bearings, notwithstanding that the materials of the *Ts* was somewhat harder than the wrought iron in the tubes.

Ratio 1/r of Length of Least Radius of Gyration	Ultimate Load P/a, in Pounds Per Square Inch				
	Fixed Ends a	When L/R > 200; b×1.72 = a	Hinged Ends b	Round Ends c	b×4/7 = c When L/R > 140
20	46,000		46,000	44,000	
40	40,000		40,000	36,500	
60	36,000		36,000	30,500	
80	32,000		31,500	25,000	
100	30,000		28,000	20,500	
120	28,000		24,300	16,500	
140	25,500		21,000	12,800	12,000
160	23,000		16,500	9,500	9,430
180	20,000		12,800	7,500	7,310
200	17,500	18,600	10,800	6,000	6,170
220	15,000	15,200	8,800	5,000	5,030
240	13,000	13,000	7,500	4,300	4,300
260	11,000	11,200	6,500	3,800	3,720
280	10,000	9,900	5,700	3,200	3,260
300	9,000	8,600	5,000	2,800	2,860
320	8,000	7,800	4,500	2,500	2,560
340	7,000	6,900	4,000	2,100	2,280
360	6,500	6,100	3,500	1,900	2,000
380	5,800	5,250	3,000	1,700	1,715
400	5,200	4,500	2,500	1,500	1,430
420	4,800	4,000	2,300	1,300	1,320

If the basis of the reasoning by which the proposed formulas are derived is correct, then the tested strength of the angle and tee struts with round, hinged and clamped ends are inter-related thru a simple constant for values which depend upon the development of the critical stress area at the point of initial tension on the convex side. This state of stress will occur approximately at a slenderness ratio of $L/R > 140$ for hinged ends and $L/R > 220$ for clamped ends. For lower values the material will likely be strained beyond its yield point value before developing this critical state of stress. Now it was assumed that for round ends the twisting and bending strains were equal and on this basis the ultimate unit stresses in *c* should equal 4/7 those of column *b* because the twisting strain in case *b* specimens is but one fourth that of case *c* specimens. Substantial agreement is disclosed by the table.

Comparing *a* and *b* specimens by virtue of the end restraint the twist is the same percent of the total in each but the capacity to resist bending is three times as great for cases (a) as case (b) hence if our reasoning be correct (a) values should equal substantially 1.72 times *b* for values of $L/R > 220$, as shown by the tabulation.

13. Pin and Square Ended Struts Compared from, the Standpoint of Detail Connections: The influence of details of the connections of pin and square ended struts on their ultimate strength and elastic behavior is a matter of practical importance. Whether the detail of the riveting to the connection renders the column rigidly fixed or restrained at the ends is a matter to be given consideration in the determination of the working stresses. Again, the diameter of the pin has a decided influence on the strength of the hinged strut. If this be investigated by approximate methods of stress analysis it will appear at once that the fiber stress area about the pin is crescent shaped which may rotate about the center of the pin with respect to the axis of the column, thereby reducing the middle ordinate of the locus of resisting compression and hence increase the relative strength of the strut.

If the diameter of the pin be not less than one third the breadth of the column, this increase may be treated for practical purposes as sufficient to warrant the value of $n = \frac{1}{2}$ in determining the reduction for column bending where the slenderness ratio is less than $L/R = 120$. In other words, with this size of pin the spread of the bearing combined with the friction at the joint is sufficient to permit the consideration of the connection as mid way between that of the knife edge and fully restrained conditions.

14. Short Columns: As the length of the column in terms of its lateral dimension becomes small the tendency toward flexure by the application of the load rapidly decreases until for a slenderness ratio of less than 30 L/R the yielding of the material determines its ultimate compressive strength, and moreover when L/R becomes less than 20 the yielding causes buldging rather than buckling of the material and then with considerable set the resistance rises far above the yield point value of the material itself. This is well illustrated by the table given by Christie, Transactions, Am. Soc. C. E. 1884, page 265, for short ended struts of angle section of iron mild and hard steel. (See following table).

In bridge and building construction as well main compression members are relatively short, for the most part between the ratios of 50 to 70 *L/R* and limited by specifications to a slenderness ratio within 120 *L/R*. Accordingly with such high yield points as silicon, nickel or other alloy steel develops an increase of thirty to forty per cent in the working stresses over those permissible for structural grade steel may be adopted.

FLAT-ENDED STRUTS

OF ANGLE SECTION

ULTIMATE RESISTANCE IN POUNDS PER SQUARE INCH OF SECTION

Length Divided by Least Radius of Gyration	Iron	Mild Steel	Hard Steel
20	49,000	72,000	100,000
30		51,000	74,000
40	40,000	46,000	65,000
50		43,000	61,000
60	35,000	41,000	58,000
70		39,000	56,000
80	32,000	38,000	54,000
90		36,500	51,000
100	29,000	35,000	47,000
110		33,500	43,500
120	26,000	31,500	40,000
130		29,000	36,500
140	23,500	27,000	33,500
150		25,000	30,800
160	21,000	23,000	28,300
170		21,000	26,000
180	19,000	19,500	23,800
190		18,000	21,800
200	16,500	16,500	20,000
210		15,200	18,400
220	14,000	14,000	16,900
230		13,000	15,400
240	12,000	12,000	14,000
250		11,100	12,800
260	10,500	10,300	11,800
270		9,600	11,000
280	9,000	9,000	10,200
290		8,400	9,500
300	7,500	7,900	9,000

15. The Elastic Strength and Ultimate Strength of a Short Column are not propotional to each other. The series of tests on wrought iron Phoenix columns presented to the American Society Civil Engineers in 1882 by Clark Reeves & Company, illustrates this fact clearly. For value of $L/R=17$ to 116, the ultimate strength decreases from over 50,000 to 35,000 pounds per square inch, while the elastic strength between these limits presents a range of hardly three percent, i.e., from 28,000 to 29,000 pounds per square inch. In other words, the elastic strength of the Phoenix column is shown to remain nearly constant while the ultimate strength increases 40 per cent between 120 L/R and 20 L/R.

The column ends were flat and may be compared to the tests of tees and angles with flat ends to investigate the relation of the symmetrical to the unsymmetrical section. The test results of Christie are tabulated at the right of the accompanying table. From it, it will be noted that the average ultimate strength of the tee and angle was 27,000 pounds per square inch against 35,000 pounds when L/R was 116. On the other hand, the strength of the tee and angle relative to the Phoenix section increases toward equality as the ratio of L/R decreases toward 20.

The permanent set or compression of the twenty five foot specimens under the ultimate load was accumulated during the application of the last quarter of the load. Resistance beyond the limit of elastic strength can be given but scant consideration in the design of a structure intended to furnish elastic resistance without set which by accumulation may end in failure under repeated loading.

Theoretical endeavor has been mistakenly directed toward the derivation of a formula which would indicate the ultimate strength under a single application of load, rather than the elastic strength which limits utility.

The ultimate strength of the Phoenix specimens has been figured by Gordon's formula in the column next to the right and it may be noted that Gordon's formula* fits closely neither the ultimate nor the elastic limits of strength of the material with constants derived for solid sections of like material.

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* By adoption of a special value of a, Mr. Buscaren fitted this formula to the test values closely for values of L/R above 30 but not below. Trans. Am. Soc. C.E. 1882. In the same paper Mr. C. L. Strobel presents a tabulation of Hodgkinson's tests from which Gordon calculated the constants of his formula.

TESTS OF SQUARE ENDED WROUGHT IRON PHOENIX COLUMNS 1881 WATERTOWN ARSENAL

No. of Experiment	Length of Column	Ratio L/R Flat Ends	Sectional Area sq. in.	Elastic Limit Lbs. per sq. in.	Ultimate Strength Lbs. per sq. in.	Total Ultimate Strength lbs. by Gordon's formula	Ultimate Strength Tees & Ls Christie Experiments
1	28′0″	112	12.062		35,150	330,146	27,000
2	28′0″	112	12.181		34,150	333,459	
3	25′0″	100	12.233	27,960	35,270	352,013	28,000
4	25′0″	100	12.100		35,040	348,119	
5	22′0″	88	12.371		35,570	372,837	31,000
6	22′0″	88	12.311		34,360	371,043	
7	19′0″	78	12.023		35,365	377,955	32,000
8	19′0″	78	12.087	29,290	36,900	380,197	
9	16′0″	64	12.000		36,580	391,701	33,000
10	16′0″	64	12.000		36,580	391,701	
11	13′0″	52	12.185	28,890	36,857	410,660	38,000
12	13′0″	52	12.009		37,200	406,866	
13	10′0″	40	12.248	26,490	36,480	423,886	40,000
14	10′0″	40	12.339	28,360	36,397	427,047	
15	7′0″	28	12.265	29,350	38,157	433,021	43,000
16	7′0″	29	11.962	29,590	43,300	469,324	
17	4′0″	16	12.081		49,500	432,132	47,000
18	4′0″	16	12.119	28,050	51,240	433,507	

NOTE:—That Phoenix column tests 1, 2, 3, 4 develope 5/4 the ultimate strength of tees and angles of the same slenderness ratios. See Art. 2 dealing with twist and following chapter discussing hardness of metal.

TABLE II--COMPARISON OF RESULTS OF TESTS OF FULL SIZE COLUMNS WITH FORMULAS OF T. H. JOHNSON AND J. B. JOHNSON

Formula No. 1 (T. H. Johnson)

For Iron: $p = 37{,}000 - 128 \frac{1}{r}$

For steel $p = 47{,}000 - 178 \frac{1}{r}$

Formula No. 2 (J. B. Johnson)

For Iron: $p = 30{,}700 - 0.51 \left(\frac{1}{r}\right)^2$

For steel: $p = 39{,}900 - 0.82 \left(\frac{1}{r}\right)$

Test	Material I = Iron S = Steel	Area of Section Sq. In.	$\frac{1}{r}$	CRIPPLING LOAD			Elastic Limit Lbs. per Sq. In.	Yield point Lbs. per Sq. In.
				By Test	By Formula			
				Lbs. Sq.	No. 1	No. 2		
1	I	17.78	98	24,800	24,500	25,800		18,000
2	I	17.56	120	26,350	21,600	23,400		20,500
3	S	15.25	83	34,270	32,200	34,300		23,600
4	S	14.15	83	32,900	32,200	34,200		24,700
5	I	25.4	43	23,290	31,500	29,800	13,800	14,750
6	I	22.8	36	28,220	32,400	30,000	13,200	15,300
7	I	27.95	39	29,210	32,000	29,900	15,200	19,700
8	I	27.95	39	29,900	32,000	29,900	15,200	17,900
9	S	14.22	97	27,790	29,700	32,200	12,600	17,550
10	I	28.38	46	26,200	31,100	29,600	16,700	18,500
11	I	28.20	46	26,230	31,100	29,600	15,950	19,500
12	I	27.46	29	30,770	33,300	30,300	17,300	20,000
13	I	27.28	42	28,200	31,600	29,800	20,150	22,000
14	I	21.78	40	26,260	31,900	29,900	17,200	20,650
15	I	30.81	37	30,300	32,300	30,000	17,000	24,300
16	S	29.36	46	30,640	38,800	38,200	25,500	28,900
17	S	15.80	45	31,680	39,000	38,200	19,000	22,100
18	S	32.11	34	31,980	40,900	38,900	22,600	26,500
19	S	32.16	35	33,950	40,800	38,900	18,700	23,300

C. P. Buchannan, Engineer of Bridges, Pennsylvania Lines West of Pittsburgh, contributed to the Engineering News, December 26, 1907, a comparison of the ultimate strength by test of a number of full size bridge members with that calculated by T. H. Johnson's formula on the one hand, and the formula of J. B. Johnson in Modern Frame Structures on the other.

Table II presents a condensed tabulation of Mr. Buchannan's tests. The sections of tests 1 to 4 were four Zee-bars with a web plate, tests 5 to 8 and 10 to 13 inclusive, 15, 16, 18 and 19, two webs four angles and a cover plate, test 17 two webs and four angles laced, test 14 two channels and a cover plate. The sections of the cover plate were unbalanced sections, a disadvantage from the standpoint of the development of the maximum elastic strength for the cross section.

For values of L/R from 35 to 40 the ultimate strength by test varied from 16 to 25 percent less than computed while for values of L/R from 80 to 100 there was closer agreement. Since the formulas in question are based on the premise that the material up to its ultimate resistance deports itself approximately in accordance with Hooke's law rather than the law of flow after elastic resistance has been passed this is not to be wondered at.

This wide divergence of the limits of elastic resistance and ultimate strength of the short column above noted is to be attributed to incipient local buckling unrelated to or independent of the flexure of the shaft as a whole. A less rigid portion of the make up by slight buckling relieves itself of a portion of its load and with increased load suffering some small permanent deformation indicated by an Increased compression of the shaft in proportion to the applied ioading and imperfect recovery on removal of the load.

The divergence of the limit of elastic resistance and the ultimate strength of the specimen is far more pronounced in the unsymmetrical section for a given ratio of L/R than in the symmetrical section.

For example in the steel angle column made up of four 6 × 3½ × 3/8 angles 2 × 3/8 lacing, 11 inch back to back, 25 feet centers of pins with one 5-3/16 pin and one 4-11/16 pin, the elastic limit in Mr. Buchannan's test was found at approximately 12,600 pounds per square inch with the development of an ultimate strength of 27,790 pounds. The ratio of $L/R = 97$. The material was Bessemer steel with an elastic limit approximately 39,000 and ultimate strength of 62,000 to 65,000 pounds per square inch.

Comparing this with the Phoenix section of iron we find an elastic limit of 28,000 pounds, an ultimate of 35,000 pounds, a difference too great to be accounted for by the difference in end conditions to say nothing of the fact that the angle section was steel and the Phoenix section wrought iron.

It will be noted that the Zee-bar column tests 1, 2 and 3, have a ratio of L/R comparable to that of test 9 of the section of four angles laced. The crippling load of steel specimens 3 and 4 was 32,900 and 34,270 compared to 27,800 for the angles. The indications of permanent set are given in the article as 23,600 pounds per square inch and 24,700 against 17,500 for the angle section and an elastic limit undoubtedly very close to 22,000 for the Zee sections against 12,600 for the angle section.

Such wide difference in the elastic limit of strength is obviously not a direct function of L/R of the section as a whole but rather a function of its make-up, the resistance to local buckling of the outstanding flange of the angle compared with the outstanding flange of the Zee section.

As the length of the column decreases below 160 L/R for hinged ends, the ultimate resistance may be reached before the critical state of stress is reached represented by a triangular compression stress area. Accordingly, the rigidity of the column under bending for a given elastic stress rises rapidly within this limit. The twisting resistance of the shaft as a whole increases in proportion to the decreased length but on the other hand, because the maximum stress is always less than twice the average compression the ability of the outstanding flange to relieve itself of part of its compression by throwing it upon the balance of the section thru twisting of the shaft as a whole decreases in the case of a laced flange and again resort to strip theory of analysis appears the only recourse.

16. The Workmanship in Fabricating the Strut made up of several parts affects its compressive strength. If the rivet holes are punched instead of drilled the metal is distorted and a state of residual strain is induced, evidenced by the curvature the section assumes after punching where the line of holes is unsymmetrical. This curvature is quite marked in angle, channel and *Z* sections and must be straightened out in assembly.

As the thickness of the metal approaches or exceeds the diameter of the punch the greatest displacement occurs decreasing rapidly as the thickness of the metal decreases with reference to the diameter of the holes.

Again, the thinner the shape rolled from a given billet the harder it is because the surface material is chilled by the water which keeps the rolls cool, thus hardening a larger percentage of the cross section the smaller the thickness and the greater the travel under the rolls.

Unequally rapid cooling of the web and flange of an *I* or *H* section frequently results in slight buckling of the web and internal compression stress of considerable intensity. Evidence of this state of compression from shrinkage strain in the thin web of *I* sections is frequently noticeable in peeling of mill scale from the surface of the web on intersecting straight lines at 45 degrees to the axis of the beam.

Residual strain from punching the thicker metal and internal stresses above noted reduce both the elastic and ultimate compressive strength of the specimen an amount which is independent of the relative length of the specimen and can therefore be separated or distinguished from the elastic phenomena of bending or twisting by noting the divergence of the curves of strength toward the extremes of slenderness ratios.

In large *cross sections* the customary laminations (plates riveted together in lieu of solid material) introduces that source of weakness discussed in Art. C.

Punching stretches the material along the lines of the holes causing an uneven distribution of longitudinal stress tending to reduce the compressive strength and tensile strength a like amount. Direct experimental evidence is meager as to the effect of punching on the compressibility of mild steel but[*] test results upon the tensile resistance of the cross section indicate that for a thickness half the diameter of the punch the net section per square inch is weakened but a small amount but where the material approaches the diameter of the punch in thickness there is a loss of strength of about [†]twenty percent for steel having an ultimate of 58,000 to 62,000 pounds per square inch and a loss of nearly thirty percent for steel having an ultimate strength of 80,000 pounds per square inch. These values are to be kept in mind in considering the strength of built up columns with different thicknesses of metal punched instead of drilled.

17. The Determinants of Ultimate and Elastic Strength may be classified under three headings in the order of their importance, (1) the form or make up of the built section, (2) the quality of the material, and (3) the workmanship.

(1) The question of form involves the slenderness ratio, and whether the metal is distributed symmetrically about a point or an irregular cross section giving rise to twisting or buckling if the relative thinness of the projecting portions and the unsupported webs are too great, and whether portions of the cross section are made up of laminated plates riveted together, and the form of bearing, whether hinged or fixed, as discussed in preceding articles.

(2) The quality of metal, whether wrought iron, mild steel,

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[*] Kirkaldy's experiment. Lewis, Engineer's Club Philadelphia, Oct. 1891.

[†] Test pieces 3" wide one hole. See also experiments by W. Parker Inst. C.E. Vol. LXIX.

hard steel or alloy steel, affects the ultimate strength differently dependent on the slenderness ratio. In the long column the strength ratio is determined largely by the modulus of elasticity for such lengths as develop the critical stress area outlined in Fig. C2. For medium lengths over 50 L/R the yield point value reduced for bending, twisting or laminated make up, determines the strength, while for slenderness ratios under 50 L/R for iron and under 20 L/R for mild steel the ultimate strength rises above the yield point value of the material.

Workmanship as it affects the strength is outlined in a general way in the preceding section but not the quantitative effect. As twisting and buckling depends on the relative thinness it is obvious that the ultimate compressive strength of the very short specimen under 20 L/R will be greater per square inch with thicker metal than the thinner sections which are harder and have a higher yield point. Again, as the thicker section has very low twisting stresses in the long column because E differs little with hardness, the heavier section would be stronger per square inch than the lighter section in which the twisting stresses, because of the thinness of the material are high compared with thicker metal.

Where the punching is symmetrical about a center, as for instance in the Phoenix column section, its effects in reducing the ultimate strength of the fabricated members would be a minimum. Where it is unsymmetrical with respect to the pieces which make up the section, the reduction in the ultimate strength will be a maximum.

Internal strain from punching and unequal cooling is reduced materially in the course of time by the disturbance of the equilibrium of internal stress thru temperature change and range of working load upon the member. See Art. 18 and 19, Chapter V, Sec. I.

The determinants under workmanship undergo a favorable modification in course of time. All others remain constant except for the effects of corrosion in reduction of section and the effect of repeated overstrain beyond the limits of elasticity of the metal.

18. The Replacement of Wrought Iron by Steel is to be studied as a historical background from which divergent ideas and opinions of today have sprung. In riveted bridge construction the use of steel was initiated on a large scale in constructing the Forth Bridge begun in 1882 and finished in 1890.

For tension members the steel was to have an ultimate resistance of not less than 30 tons (67,200 lbs. per square inch) nor more than 33 tons (73,900 lbs. per square inch) with not less than 20% elongation in eight inches. For compression members not less than 34 tons (76,160 lbs. per square inch) nor more than 37 tons (82,880 lbs. per square inch) with not less than seventeen per cent elongation in eight inches. Quoting Mr. Westhofen, London Engineering 1890:

> "The choice of material for constructing a bridge of such novel design and extraordinary magnitude must have been the subject of much anxious thought and reflection to the engineers . . . there can be no two opinions that the choice was a happy one." **This steel was subjected to every conceivable test both in a scientific manner for** purposes of research and investigation and in an unscientific manner by workmen whose only excuse can be that they knew no better . . . A more uniform, homogeneous, and satisfactory material could not be wished for." "To quote one instance, pieces of scrap picked up at random drilled ¾ inch deep when the machine was stopped gave a single cork screw shaving a yard in length starting from the moment the drill touched the steel and attached yet by the end to the piece of scrap out of which it was bored." . . . This behavior of the steel had a great deal to do with the confidence the workmen regarded every portion of the structure and with their belief that no possible load they could pile on the temporary platform could bring about a collapse."

The diameter of the main vertical posts was 12′ 0″ and unsupported length 150 feet or a ratio of L/R of 38 roughly. For the eight foot diagonal tubes this ratio is increased to 60. A working stress on these members was figured at 7½ English tons or 18,800 lbs. per square inch with no reduction for column bending, etc. The ultimate strength was estimated at 22½ tons or 50,400 pounds per square inch which appears very conservative in the light of Dagrons tests of laced steel columns of like hardness to be discussed later. This value seems on a par with the strength developed in nickel steel columns of American design lacking the satisfactory disposition of metal adopted by Messrs. Fowler and Baker. That as pioneers in the use of steel in lieu of wrought iron these engineers treated the material on its merit unprejudiced by working stresses common for iron in use at that date is as surprising as the general failure of the engineering profession to profit by their example.

For the tension members they selected that grade of carbon steel possessing the maximum strength consistent with toughness while for the compression member they recognized that to develop correspondingly high compressive strength without yeilding a higher degree of rigidity or hardness of the material is a necessity.

The use of steel in 1884 was decided upon in the construction of the Susquehanna River Bridge on the Philadelphia branch of the Baltimore and Ohio Railroad and lack of data led to the experiments described by J. G. Dagron in Trans. Am. Soc. C. E. Vol XX, June 1889.

The specification for the steel required an ultimate strength of not less than 80,000 pounds per square inch, a yield point value not less than 50,000 pounds, an elongation not less than 15% in eight inches, a reduction of area not less than thirty percent at point of fracture and that a specimen 3/4″ round should bend cold 180 degrees about its diameter without crack or flaw.

There were eight test columns in all, six were made up of two built channels 8″ back to back 8″ × ¼″ plates and 2¼ × 2¼ × ¼ angles with 5/8 soft steel rivets 4″ centers laced with 1¾ = ¼ bars 9″ alternate spacing. Pins were 4-3/8″ parallel to the webs. Lengths center to center pins 16 feet, 20 and 24 feet, Slenderness ratios *L/R*, 42 to 63. Ultimate strength 41,000 pounds per square inch for the shorter to 40,000 pounds per square inch in round numbers for the longer columns. Test specimen gave an ultimate strength of 83,000 to 84,000 pounds per square inch. Elastic limit, i.e. yield point 51,000 to 53,000 pounds per square inch. Elongation 19 to 20% in eight inches.

The ultimate strength was so much less than expected that it was commented upon by Mr. James Christie in the discussion of the paper as follows:

> "Experiments on the compression of solid bars of wrought iron or steel, having the same slenderness ratios, i. e. (42 to 63) as the column tested by Mr. Dagron usually gives results for maximum resistance at least equal to the elastic limit of the material, steel bars averaging 10 percent and iron bars about 25 percent above the elastic limit." "The experiments on these built columns show a reduction of the maximum resistance below the elastic limit of 25 percent."
>
> "The resistance of these columns was only five to ten percent greater than we would expect from similar columns of iron having an elastic limit of 30,000 pounds."

Now in Art. 20, we have recorded the test results on Phoenix square ended columns which run but five percent lower in ultimate strength than these steel specimens. For tests of pin end conditions we may refer to the laced iron channel columns tested by the Detroit Bridge and Iron recorded in Lanza's Applied Mechanics:

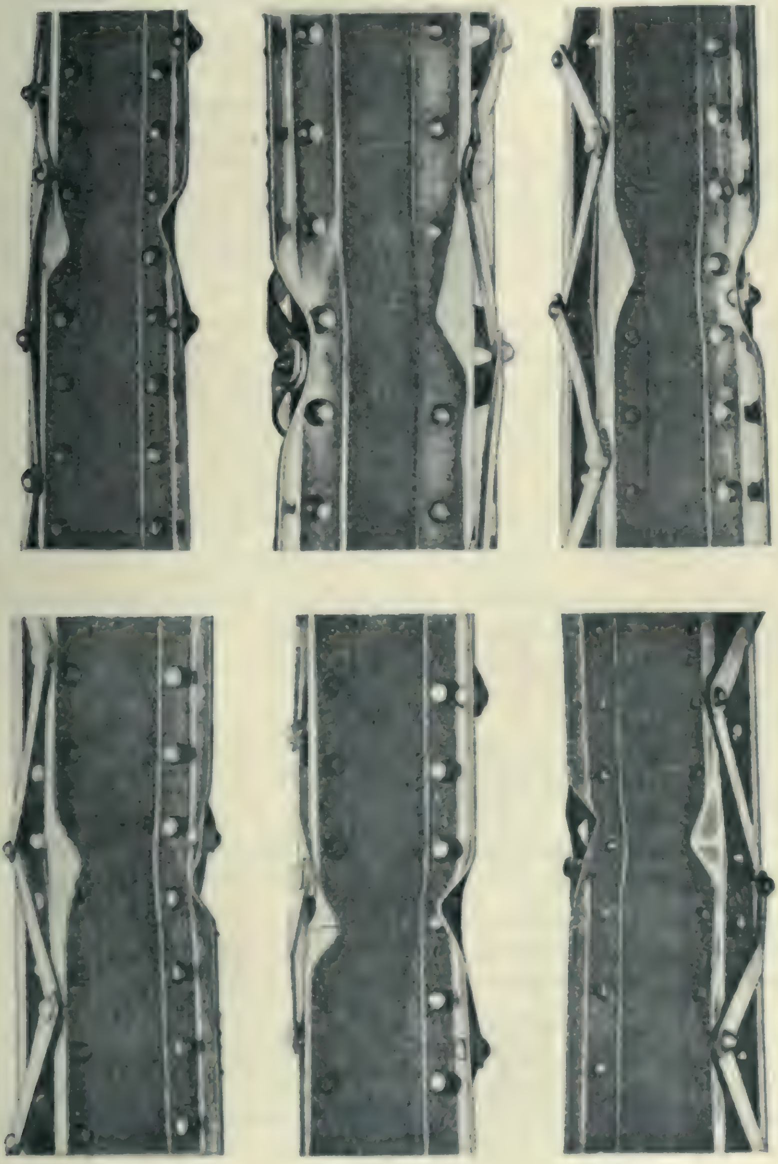
*Ends	Channel Bars	Length	Section	L/R	Lattice Spacing	Ultimate Strength
Pin Perpendicular to web	8″	13′4″	7.5 sq. in.	53	18″	35,000 aver of 2
	10″	20′10″	9.8 sq. in.	68	22″	33,700 aver of 2
	10″	16′8″	9.6 sq. in.	56	22″	33,900 aver of 2

*Tests with pins parallel gave identical results for like ratios of L/R, Watertown Arsenal, 1883.

DAGRON TESTS OF LATTICED COLUMNS. PIN ENDS

From Trans. Am. Soc. C. E. 1889

Showing detail buckling of angle flanges at failure. Failure by bending parallel to pins which were parallel to the web. All failures occurred near the center of the length.



These tests show strengths for laced wrought iron channels running fifteen percent under the hard steel values and but five to eight percent above the yield point value of iron taken at 31,000 pounds per square inch instead of the twenty five per cent that Mr. Christie gives for solid bars of iron or like slenderness ratios.

In seven out of the eight tests described by Mr. Dagron failure occurred near mid length of the column, thru initial buckling of the outstanding angle flange between lattice connections.

Consider a strip at the outer edge of the flange; as the angle is ¼ inch thick and it is 18 inches center to center lattice connections $L/t=72$ and L/R for the strip$=210$.

Now if this outstanding strip were unrestrained by the angle at the root it would fail at about 12,000 pounds per square inch, taking n at ½ in the curves of Fig. C6.

In the test of tees and angles the reduction in strength for pin and square ends was one fourth for a flange projecting six times its thickness. This flange projects 7 thicknesses beyond the C. G. of Channel so that the reduction in ultimate yielding would be $7^2/6^2 \times (41-15) \div 4 = 9.8$ kips. (1000 lb. units.)

If now we add 10% to the yield point value of 53,000 and deduct 9,000 for bending (Fig. C6) and 9,800 for buckling stress we arrive at the approximate value of the ultimate strength of the test.

Continuing our historical review, in the Phoenix handbook of 1890 we find specification of STEEL embodied in three lines, sandwiched into a long specification for wrought iron. (Page 118 Phoenix Handbook 1890) STEEL:

> "15. No specific process or provision of manufacture will be demanded, provided the material fulfills, the requirements of this specification." (i.e. for wrought iron).

From the manufacturers effort to measure the new structural metal by the yardstick of the old claiming for it far greater toughness and ability to stand more severe cold bend and drifting tests while possessing greater strength, uniformity and homogeneity arose that type of specification which soft steel alone might fill.

The terms TOUGH and SEVERE in respect to a test in their proper significance should clearly define a superior product from the structural standpoint but unfortunately the characteristic of toughness has been confused with the attributes of weakness. The spectacular nature of a drift or bend test has been confounded with the severity of the test. Let us refer to Webster for the correct meaning of the term tough and we find that from the physical standpoint it represents that which is strong, stiff, rigid, not flexible, as a tough bow presenting the property yielding to force without brittleness. In other words, toughness is measured by the degree of strength and rigidity, exhibited under the applied force coupled with ultimate failure without manifest brittleness. A severe test according to Webster is one difficult to endure, hard or rigorous involving the work function or quantitative expenditure of much mechanical energy in a distortion test.

Mr. Harry J. Lewis, Engineering News, April 1895, has shown soft steel to be capable of drifting a 13/16" hole in a 3/8" plate to eight times its area and a 3/4" hole in a 1-3/8" bar to six times its area and of being bent double over on itself 180 degrees again and again in the same bar without cracking.

These characteristics of soft steel are excelled greatly by lead in the ease by which a hole in the sheet may be enlarged by drifting or the material doubled over upon itself but it is not a characteristic of toughness nor severity of the test if we measure the work producing the distortion in terms of units of distortion.

The mechanical energy required to distort the metal a given amount is a measure of its toughness providing it does not exhibit brittleness of sudden failure without distortion. The measure of structural utility of the material lies in its strength and stiffness and its [*]toughness is measured by the amount of force or energy required to distort or deform it a given amount.

Considered from this angle the grades of steel selected by the engineers of the Forth Bridge appear to possess for carbon steel the characteristic of maximum toughness and ability to withstand the most severe drifting test consistent with economic strength for the tension member and the greatest resistance to plastic deformation in the compression member to be secured in carbon steel consistent with toughness.

The early classification of steel was SOFT from 50,000 to 58,000 pounds per square inch, MEDIUM 60,000 to 70,000 pounds per square inch, INTERMEDIATE from 70,000 to 80,000 pounds per square inch and hard above 80,000 pounds per square inch.

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[*] A practical conception of relative or proportionate toughness can best be acquired by performing the following experiment: Secure two 5/8" round bars 20" long one of mild steel, ultimate about 50,000 lbs. per square inch and another silicon steel low in phosphorus, sulphur and manganese having an ultimate strength of 125,000 pounds per square inch and bend each with a hammer or sledge 180 degrees to a radius equal to the diameter of the bar. The student's mental estimate will undoubtedly be that it requires a thousand fold more muscular energy to bend the latter than the former though a few hundred fold difference in mechanical energy would be nearer the truth.

C. C. Schneider adopted a working stress in tension 15,000 pounds per square inch for soft and 17,000 pounds per square inch for medium steel. For compressive strains

$$\text{For soft steel, } p = \frac{15{,}000}{1 + \dfrac{l^2}{13{,}500\, r^2}}$$

$$\text{For medium steel, } p = \frac{17{,}000}{1 + \dfrac{l^2}{11{,}000\, r^2}}$$

p = permissible working strain per square inch in compression.

l = length of piece in inches, center to center of connection.

r = least radius of gyration of the section in inches.

See Pencoyd Handbook, 1898 and earlier editions.

The divergent values for which these specifications were adopted were obviously intended to cover the average resistance of soft steel at 55,000 pounds and medium steel at 65,000 pounds per square inch. But the manufacture of two different grades of steel for the same class of structure to meet the views of those who wanted a material which would stand excessive distortion in drift tests and bend tests and those who wanted structural stability and economic hardness on the other hand caused the steel corporation to favor a modified specification in which the ultimate strength limit of soft steel was increased to 62,000 pounds per square inch to overlap the medium steel tolerance with elongation 25 percent, bending test 180 degrees over on itself; for medium steel 60,000 to 70,000 pounds, elongation 22 percent bending test 180 degrees to a diameter equal to the thinness of the piece tested.

Now the tested ultimate strength is higher the more rapidly the load is applied and with a uniform charge and variable speed of test it is not difficult to keep within a range of about 2,000 pounds per square inch so that with this over lapping of the different grades the rolling mill could supply soft steel grade and medium grade steel from the same melt, poured from the same laddle, rolled from the same ingot and the professional bridge engineers of the country at large were allowing one eighth increase in the working stress on their bridge members upon no more substantial basis than the label of the mill product as medium steel which in point of strength qualified only within the lowest limit of tolerance permitted under the specification.

The absurdity of the situation seems to have been appreciated for in the next edition of the Carnegie Handbook we find the recommendation for bridge steel from 55,000 to 65,000 pounds per square inch, a material the lower range of which is much too soft and the upper range insufficiently hard to secure the most economic results as indicated by the demand for harder material and the increasing use of expensive alloy steel to secure economic hardness in the make up of large compression members.

19. The Zee Bar Column furnishes an interesting study of the effect of change of proportion upon manner of failure and relative ultimate strength.

This type of section was introduced by Mr. C. L. Strobel in 1886 in a bridge at Kansas City for the Chicago, Milwaukee and St. Paul Railroad. Zee bar columns in the bridge were made up of four 3″ × 5″ × 3″ Zee-bars 11/16″ thick. Column sections tested by Mr. Strobel at Keystone Plant were made of four 2½″ × 3″ × 2½″ Zee-bars 5/16″ thick. Instead of connecting the Zee-bars by a plate they were laced together in the middle plane and at each end there were three tie plates, one in the center and one on each outside face. The specimens were square ended and the failure in each case was by simple bending.

The accompanying table* gives the length, sectional areas, elastic limit of the column, ultimate strength per square inch, ratio of length to least radius of gyration, ultimate strength by Rankine-Gordon formula and the ultimate strength by Strobel's formula.

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*Data from Vol. XVIII, Trans. Am. Soc. C. E. 1888.

GENERAL RESULTS OF COMPRESSION TESTS OF 15 Z-IRON COLUMNS

RADIUS OF GYRATION, LATTICE BARS NOT CONSIDERED, = 2.05.

No. of Column	Length of Column	Sectional Area Sq. Ins.	Ultimate Strength		Ratio of Length to Least Radius of gyration	Ult. Str. By formula (Rankine-Gordon) $\frac{36{,}000}{1+\frac{l^2}{36{,}000\ r^2}}$	Ult. Str. by formula 46,000—125 L/R
			Elastic Limit Total Pounds	Ultimate Strength Pounds per sq. in.			
3	10′11¼″	9,435	28,800	36,800	64	32,300	
4	10′11¼″	9,985	26,400	34,600	64	32,300	
2	15′0″	9,480	27,400	34,600	88	29,600	35,000
5	15′0″	9,280	26,000	36,600	88	29,600	35,000
1	19′0¾″	9,241	27,000	33,800	112	26,700	32,200
6	19′0¾″	10,104	22,000	33,700	112	26,700	32,200
7	22′0″	9,286		30,700	129	24,600	29,900
8	22′0″	9,286	26,000	29,500	129	24,600	29,900
9	22′0″	9,286		30,700	129	24,600	29,900
10	25′0″	9,156		28,100	146	22,600	27,750
11	25′0″	9,456		28,000	146	22,600	27,750
12	25′0″	9,516		28,400	146	22,600	27,750
13	28′0″	9,375	23,600	27,700	164	20,600	25,500
14	28′0″	9,643	20,000	28,000	164	20,600	25,500
15	28′0″	9,375		27,600	164	20,600	25,500

The elastic limit was taken at the commencement of permanent lateral deflection or permanent set or contraction.

It will be seen from these values that the Z-bars arranged in this way give ultimate strengths approximating those of the Phoenix column but that the elastic limit for the higher slenderness ratios is of a lower order.

> Quoting Mr. Strobel:
>
> "The values obtained are near approximations to the Watertown results with Phoenix columns, and exceed those heretofore obtained with other types of columns. This favorable showing for the Z-iron columns should probably be attributed to the fact that the material in the outer periphery of the cross section, on which dependence must be placed to hold the column in line, is not weakened by rivet holes, but is left solid and unbroken, and is, therefore, in the best shape to do its work effectively."
>
> "The average of Hodgkinson's cylindrical tubes, for lengths ranging from 40 to 90 radii (14 to 32 diameters) was 33,100 pounds per square inch. (A softer iron)
>
> The average for the Watertown tests of Phoenix columns for lengths ranging from 40 to 110 radii (14.1 to 39.5 diameters) was 35,800 pounds per square inch.
>
> The average for Mr. Bouscaren's tests of latticed channel columns

> for lengths ranging from 59 to 90 radii (21 to 32 diameters) was 32,300 pounds per square inch.
>
> The diameters referred to in the above are the diameters of an equivalent cylindrical tube whose area is concentrated in the circumference and whose radius of gyration is the same as that of the section considered.
>
> The present tests on Z-iron columns show no variation in the ultimate strengths for columns 64 radii (23 diameters) and 88 radii (31 diameters) long, the average being 35,650 pounds per square inch.

The iron from which the test specimens were made was of excellent quality from 49,000 to 52,000 pounds per square inch ultimate having an average yield point value of 32,000 to 35,000 pounds per square inch and an elongation from 13 to 22 percent in eight inches.

Tests of square ended Z-bar columns of steel having an ultimate strength of 58,000 to 59,000 pounds per square inch and a yield point value of 38,000 to 39,000 pounds tested by the Bureau of Standards, gave an average ultimate strength for a slenderness ratio of $L/R = 50$ of 35,700 pounds; for $L/R = 85$ an average value of 32,800 pounds; for $L/R = 120$ of 29,700 pounds for metal $\frac{1}{4}$ inch thick and for heavier metal 5/8 inch thick of 27,300 to 32,900 pounds for the smaller slenderness ratio.

It appears therefore that the wrought iron columns tested by Mr. Strobel gave values for a slenderness ratio of $L/R = 64$ about five percent higher than the steel column. And for a slenderness ratio of 120, six to seven percent higher than steel having an ultimate strength from 16 to 20 percent higher and a yield point value about fifteen percent higher than the wrought iron.

Is this difference in favor of the wrought iron to be attributed to compressive strength of wrought iron superior to steel or to the form of the section? The final conclusion of the special committee of the American Society, Vol. LXXIII, 1919, was that the form of the section, whether box, H, Z, Channel or I, or four angles and plates, made little difference for slenderness ratios of 50 to 85 L/R, but according to their progress report published in Trans. Am. Soc. C. E., Vol. LXCI, 1910, the adjusted value of the ultimate strength of the wrought iron column to correspond to the steel should be increased twenty percent over the test results for the iron section. Consequently if the special committee were correct in their figures in 1910 their conclusion in 1919 with respect to form is from thirty to thirty five percent in error for their conclusions do not agree with the facts.

Perhaps both conclusions of this special committee, after nine years of study may be considered erroneous as indicated by the manner of failure or deformation of the steel columns when contrasted with the deformation of the wrought iron columns tested by Mr. Strobel. Mr. Strobel's column failed by simple bending, buckling in the weakest direction. About that axis the radius of gyration was about two inches while about the normal axis the radius of gyration was about twice as great.

Contrast this deportment with that of the steel columns tested at the Bureau of Standards in which the failure was accompanied by a twist or corkscrew like type of yielding. In Mr. Strobel's tests the slenderness ratio at right angles differed by one hundred percent. In the Bureau of Standards tests the slenderness ratio about rectangular axes was much nearer an equality more conducive to twisting deformation. In the details of the Strobel column the three tie plates tended to equalize the pressure over the cross section at the end preventing incipient buckling from inequalities in distribution of the load and this element of strength was lacking in the Bureau of Standards Tests which will now be considered more in detail.

20. American Society Committee Report on the Working Values of Columns and Struts was based on tests made at the Bureau of Standards and the consideration which this Committee gave the subject during the ten years following 1909. The intention of the Committee was as far as practical to eliminate all the variables except that of form. To consider thick and thin material in the make up of the sections for slenderness ratios varying from 50 to 120 L/R. The tests embodied riveted *H* sections, (four angles and a web plate); box sections, (two channels and two cover plates); two channels turned in riveted to the back of an I-beam; Z-bar sections of four 4 inch by ¼″ *Zs* with a 7 by ¼″ plate; four Z-bars 5/8″ with a 7 by 5/8″ web plate; 10″ *Is* with 11″ cover plate riveted to each flange; angle box sections similar to the channel and plate sections and Bethlehem and Carnegie light and heavy 8″ rolled *H* sections; and one sample of four bulb angles and a plate in place of four ordinary angles.

The desired physical properties of specimen tests were ultimate strength 60,000 pounds per square inch with allowed variation 1,500 pounds either way; yield point desired 38,000 pounds per square inch with 1,000 pounds variation; elongation desired 28 percent; reduction of area 56 percent; cold bend 180 degrees flat on itself.

In order that the question of the form of the section only should govern it was obviously necessary to secure material of a high degree of uniformity. In general the trend of these experiments seemed to indicate that for some cause to be determined, the sections with the thinner material gave higher ultimate strength values than the sections with thicker material in apparent conflict with the law of twisting or buckling discussed in previous articles. How much of this difference may be attributed to lack of uniformity of material and how much to the effect of shop work as between the thick and thin metal is a matter which we will commence to investigate by considering the plain rolled sections on which no punching or riveting was done. The accompanying table gives the size, length to the nearest inch, slenderness ratio, and ultimate strength developed.

TABLE A

DATA FROM TRANS. AM. SOC. C. E. 1919

Shape	Weight	Length	L/R	Column Average Ultimate Strength	Tensile Ultimate of Specimen Test	Tensile Y. p. of Specimen Test
Light Bethelhem 8″ H	32 lbs.	8′3″	50	38,000	58,900	web 37,000
	32 lbs.	14′0″	85	34,300	64,000	Rt. 38,000
		19′10″	120	32,000		
Light Carnegie 6″ H	23.8 lbs.	6′0″	50	31,000	no data	
		10′3″	85	30,400		
		14′6″	120	27,200		
Light Carnegie 8″ H	34 lbs.	7′9″	50	33,500		
		13′3″	85	31,700		
		18′8″	120	29,200		
Heavy Bethlehem H	62 lbs.	8′8″	50	35,400	57,000	32,500
		14′10″	85	32,300	to	33,700
		20′11″	120	30,000	58,000	
Extra Heavy Bethlehem H 8″ × 91 lbs.	91 lbs.	9′0″	50	25,200	Aver.	18,000
		15′4″	85	23,500	56,000	
		21′8″	120	21,300		

In considering this table the specimen tests gave ultimate strength of strips cut from the web, root and flange and a useful limit point which differs from the primitive yield point by the amount the primitive yield point may have been raised by repeated runs of the loading but may serve as a comparative basis to like advantage. It is marked *Yp* under the specimen tests.

The sections compared were light Bethlehem *H* weighing 32 pounds, Carnegie section 34 pounds per foot, with a heavy Bethlehem *H* section 62 pounds a foot.

Comparing the Bethlehem heavy *H* section with the Bethlehem light section, it will be noticed that the metal in the light section was higher in ultimate strength and in yield point value. As to the Carnegie sections no data is given. Comparing the four the Bethelhem 62 pound section is stronger than the Carnegie lighter section in the ultimate per square inch and under runs the Bethlehem light section by a much smaller amount per square inch in ultimate strength than the difference between the specimen tests in favor of the lighter section.

From these four it is obvious that as far as the properties of the material are comparable the heavier section shows relatively the greater strength as would be expected from our theory of twisting.

Considering now the specimen tests *B* the extra heavy Bethlehem *H*, 1¼″ thick, the low ultimate and yield point values of the specimens indicate a softer material finished at too high a heat.

It is noticeable that the difference between the unit ultimate strength of 50 and 120 L/R of the thick section is less than three fourths that of the thinner sections and were the length of the respective sections decreased to 20 L/R the ultimate unit strength per square inch of the heavier section would be greater than that of the lighter.

The difference in compressive strength between thick and thin specimens rolled from the same melt may be more readily accounted for by the difference in the temperature of the metal at the time of the final pass and the amount of quenching which the surface of the specimen has been subjected to in its travel thru the rolls by the water which keeps the rolls cool. If the metal is finished at too high a heat it may be soft, coarse in texture and lacking in the strength and toughness of the shapes finished at the lower and more favorable temperature.

In general the thinner the piece the greater the number of passes it has been subjected to in rolling and the greater the amount of quenching its surface has received. Quenching raises the ultimate strength more relatively than the increase in the yield point value so that for different thicknesses of material of the same chemical composition it is not possible to maintain a fixed value for the ultimate strength and also the yield point limit. They will increase or decrease under the conditions noted disproportionately. The addition of carbon under like physical treatment tends to increase the ultimate strength and the yield point limit approximately proportionate amounts, for structural steel grade.

From the wide divergence in the character of the material as indicated by the specimen tests the tests of the *H* sections affords only an indication of the general laws of change of strength with thickness where the form is proportional or similar.

TABLE B

TEST OF COLUMN SECTIONS SQUARE ENDS

(Data from Trans. Am. Soc. C. E. Vol. LXXXIII p. 1583)

Type	Section 4 Ls 5x3 16″ pl.	Length	L/R	Specimen—*Yield Point (Tension)		Ultimate Column Strength Average (Compression)	
				Light	Heavy	Light	Heavy
1	Metal 5/16	3′9″	20			36,400	
	Metal 5/8	3′11″	20				44,400
and	Metal 5/16	9′4″	50	28,800		32,700	
	Metal 5/8	9′10″	50		24,700		29,700
1A	Metal 5/16	15′10″	85	27,800		31,200	
	Metal 5/8	16′9″	85		25,300		28,100
Rivets	Metal 5/16	22′5″	120	26,600		28,300	
	Metal 5/8	23′7″	120		23,300		25,400
3/4″ ..	Metal 5/16	28′11″	155	25,200		26,200	
	Metal 5/8	30′6″	155		21,900		22,700

Type	Section 44″ Zs 7″ pl.	Length	L/R	Specimen—*Yield Point (Tension		Ultimate Column Strength Average (Compression)	
				Light	Heavy	Light	Heavy
8	Metal 1/4	10′3″	50	31,300		35,700	
and	Metal 5/8	10′6″	50		29,000		32,900
8a	Metal 1/4	17′6″	85	31,000		32,800	
Rivets	Metal 5/8	17′9″	85		29,700		31,400
3/4″	Metal 1/4	24′7″	120	28,200		29,700	
	Metal 5/8	25′1″	120		25,500		27,300

Table B shows the test results on *H* sections made up of four 5 by 3″ angles and a 6″ plate with light sections of 5/16″ metal and 5/8″ for the thick sections. These tests were extended to the slenderness ratios of 20 *L/R* and the higher ratios of 155 *L/R*. For 20 *L/R* the 5/16″ metal column tests gave a unit ultimate strength of 36,400 pounds while the 5/8″ metal gave an ultimate strength of 44,400 pounds, thus exhibiting a twenty five percent increase in resistance to buckling of the 5/8″ metal as against the 5/16″ metal. But notwithstanding this increased resistance to buckling the thicker sections did not develop as high ultimate strength as the thinner for ratios of 50 to 120 *L/R*.

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*So-called useful limit point differing from the drop of the beam as explained p. 1618 of Committee report defined as the point determined by drawing a tangent to the stress strain curve having a slope half that of its straight or nearly straight portion.

The manner of fabricating the column furnishes a partial explanation of this anomalous result. They were extra well riveted in order to make the angles and plate act as nearly as possible as a unit but in punching holes four diameters of the rivet apart in one leg of the angle only it tends to curve or bend the angle by stretching the three inch leg and compressing the five inch leg. This curvature of the angles would have to be taken out in assembling and the result would be a relatively high degree of internal stress representing the difference between drilling the holes and punching. This phase of the shop work might have been investigated by drilling part of the sections and punching a part instead of fabricating all in the same identical manner.

The magnitude of the internal stress produced by the punching an unsymmetrical line of holes depends on the thickness of the metal relative to the diameter of the punch. With 5/16″ metal and ¾″ punch the effect of the distortion of the metal between the punch and the dye might not be noticeable in the curvature of the angle but with the 5/8″ metal it would be material—readily observable.

The steel sections in the Bureau tests under run the ultimate strength developed by wrought iron Z-bar columns tested by Mr. Strobel as tabulated in Art. 18. Thus the highest compressive value for steel is 35,700 pounds ultimate strength for 50 *L/R* against 36,800 for 64 *L/R* in Mr. Strobel's tests of wrought iron while for 85 *L/R* the ultimate strength of the steel was 32,800 for the light section, 31,400 for the heavy section against a mean of 35,600 for the wrought iron. The steel columns exhibit strength inferior to that developed by the wrought iron altho specimens of the steel developed 60,000 pounds tensile resistance against 49,000 to 50,000 pounds per square inch for the wrought iron indicates an effect upon the ultimate strength of material importance produced by end details and difference of distribution of the metal in the form of the section.

Thus, the wrought iron sections of Mr. Strobel had a slenderness ratio of a much lower order about an axis normal to the plane of the lacing or web than the steel columns tested by the Bureau. In the Bureau tests the failure of the steel Z-bar column was of the cork screw type. The failure of the wrought Z column in Mr. Strobel's tests was that of simple bending. The wrought iron column had a polar moment of inertia much greater than the steel column. Hence the tendency of the steel sections to twist was reduced in the wrought iron column with the increase of the polar moment of inertia. Again, thru its three tie plates at the ends, a fairly uniform distribution of the applied loading across the area of the section of the wrought Z-column was insured. The plain milled end of the steel column without tie plates stiffening the outer flanges of the Z would be affected by any inequality of the application of the pressure which effect would not in turn be redistributed by a base detail such as practical column is fitted with. The omission of a web plate and the substitution of lacing to perform the function of stiffening the section of the wrought iron column constitutes a redistribution of the metal further from the center of the section producing more favorable results than the minimum slenderness ratio of L/R would indicate.

But the ultimate strength is only one side of the elastic deportment which we are investigating. Comparison of the superiority of the Phoenix section, symmetrical about its center, with the Z-bar unsymmetrical about its center, in the tests tabulated in Art. 14, with those tabulated in Art. 18 teach an important lesson in column design.

(a) From these tests it appears steel columns of small size of sections unsymmetrical about a center are not as strong as wrought iron columns such as the Phoenix which are symmetrical, a deficiency caused by difference in form.

(b) That where this ratio is less than 30 the ultimate strength of heavy sections is greater than that of light sections but the development of this excess strength is accompanied by set.

(c) That the average superiority in ultimate strength of the lighter built section notwithstanding a greater tendency because of lack of thickness of metal to buckle and twist is accounted for by greater hardness of the thinner metal in part and in part by the higher intensity of internal stress in the thicker metal caused by punching.

The residual strain from punching normal to the axis affects tensile and compressive resistance of the short specimen a like amount but when the axial stress is not distributed uniformly as in the column approaching rupture this reduction in ultimate resistance decreases as the slenderness ratio increases. Hence the relative weakness from this cause should exhibit decrease in the relative difference between the strength of the heavy and light sections as the length increases until at 200 L/R the heavy sections would develop uniformly greater unit strength than the light sections.

Thus the light section of 2—8″ channels 11.25 pounds and 1—8″ I 18# type 7 of Fig. C6 developed 24,000 pounds per square inch as maximum ultimate strength or no more than the fifty percent heavier section when for each L/R was increased to 155; whereas for 50 L/R the light section developed 36,900 pounds per square inch and the heavy but 29,100 pounds per square inch.

For a slenderness ratio of 50 L/R the light Z-section developed an average ultimate of 35,900 pounds per square inch and 29,700 pounds for 120 L/R while the heavy section developed 32,900 pounds average for 50 L/R and 27,300 for 120 L/R. A similar trend appears in the tests for the built box sections.

Reduction of the working stress for the steel column in building work from 14,000 maximum of the usual code allowance to 12,000 pounds per square inch (a wrought iron working stress) was recommended by the special committee in its final report of January 16, 1918. This recommendation contrasts strongly with that of an association of fabricators who would raise the present working stresses to 18,000 lbs. per square in divided by $(1 + l^2/ 18{,}000r^2.)$

General experience warrants the present practice as conservative for medium steel, and a searching inquiry into all the conditions and details of text results upon which present practice is questioned is in order, if the trouble may not be attributed to the use of a softer grade of metal than that customary in earlier practice.

In theory our committee proposed to eliminate all variables except that of form; in practice they included with variation in form the variable effects of shop work and variable hardness of metal without effort at separation. In theory they proposed taking photographs of the detail failure of the specimens which is most important data from the analytical standpoint but in practice of their final report no views are to be found and we are left to supply this deficiency as best we may from our recollections of the appearance of the scrap pile and the views published in Engineering News, June 28, 1917.

21. Columns Tested for the American Railway Engineering Association at the Bureau of Standards were built of open section consisting of two channels laced or battened. The purpose was to determine strength values applicable to forms most commonly used bridge structures. Base details were such as are used in practice in contrast to the milled end of the tests discussed in Art. 19, but the material was substantially of the same grade as far as might be secured by the same specifications.

For better comparison with the tests of the preceding article the data of the former is presented in TABLE I and for the latter in TABLE II reproduced from Engineering News, together with the cut showing six views of the manner of failure under excess deformation.

Intergral buckling of the columns was by far the most common manner of failure, not merely at maximum load but even under the additional compression. The curve assumed by the column in such case is exemplified by the photographs *B* and *F*, Fig. 1. Both the solid section columns and the latticed columns failed by bodily buckling. None showed evidence of local failure (as at the end connection wing plates, in the lattice, or in the unsupported lengths between lattice bars) prior to the point where maximum load was reached. Further, most of the columns when forced far beyond the failure point buckled into a smooth curve, showing that they developed the full efficiency of the column.

While local or detail failure was rare, some of its manifestations were of noteworthy character. The tie-plated columns of the A.R.E.A. series failed in most cases by buckling between the batten plates. That they did not develop the strength of the metal in full degree is proved more definitely by the ultimate strength (Table II), which show surprisingly low figures for all these columns. On the average, the tieplated columns were only half to two-thirds as strong as latticed columns of equivalent size and length. But as explained in a footnote to Table II, the tie-plate spacing was in all cases so wide as to make the slenderness-ratio of the individual channel between tie-plates much larger than that of the whole column.

The Phenomena of Rotary Buckling: All the Z-bar columns of the thin sections failed in the curious manner shown by *A* in Fig. 1. The middle rotated around the column axis, while the ends (held by the machine heads) stayed in original position.

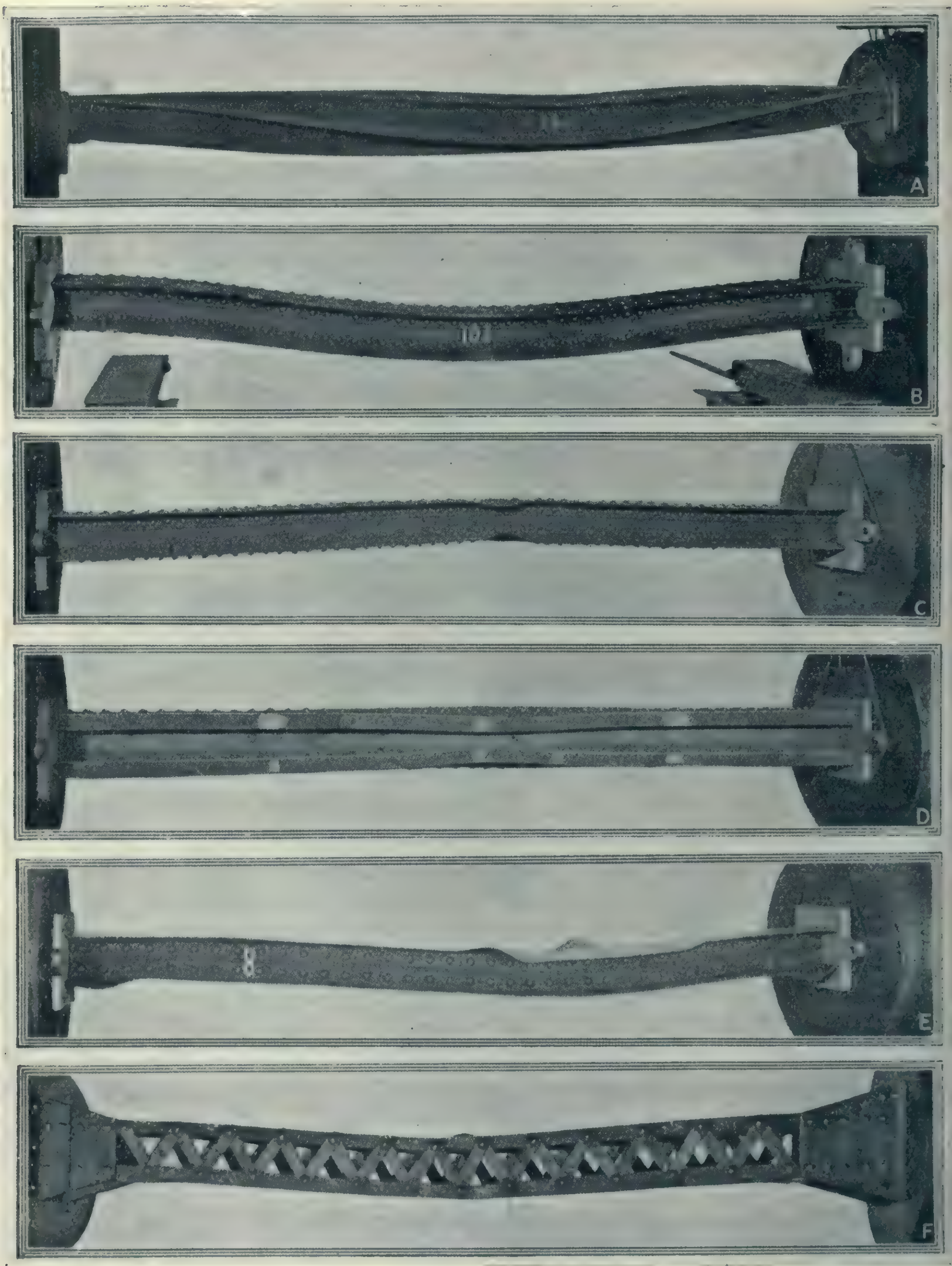

Fig. I
VIEWS OF DETAIL FAILURE OF TEST COLUMNS

This twisting appeared to be a rotary buckling. The four outstanding legs of the Z-bars tend to buckle, and this tendency produces a tangential reaction around the column axis as center of twist. If two of the four Z-bars should tend to buckle in one direction, and two in the other, balance might exist; in all other cases the net result would be a tendency to twist the middle part of the column relative to the ends. When the twisting moment becomes great enough to overcome the torsional resistance of the column core, it would produce rotation like that actually observed.

The noteworthy fact is that all the light section Z-bar columns, without exception, failed by twisting after passing the ultimate strength, while all the heavy sections Z-bar columns buckled bodily by flexure without twist.

That these light steel Z-bar columns twisted like cork screws under excess compression whereas the light Z-bar columns of Mr. Strobel's tests did not, show that when the sections are light twisting occurs when conjugate radii of gyration are equal and does not occur when the radii at right angles are very unequal. The simple bending of the heavy section in contrast to the twisting of the light sections justifies the analysis previously proposed.

The views *C D E* of Fig. 1 exhibit types of localized buckling. How much influence this local weakness had upon the ultimate strength is not quantitatively known from the tabulated results. The fact that the solid specimen fails at much higher values than the yield point and the built up or unsymmetrical solid such as the *L* or *H*[rolled sections is not sufficient proof of its material effect.

Such detail buckling as exhibited at the bases of *A* and *E* in Fig. 1 would be eliminated by the practical shoe of *F* or its equivalent.

TABLE II shows that notwithstanding the secondary action of the lacing the laced columns with the shoe developed much greater strength than the box column without a substantial base detail.

The failure indicated by *F* of bending in a plane parallel to the lacing instead of parallel to the web indicates that for equal strength at right angles with fixed ends the radius of gyration should be much greater, preferably twice as great, about an axis parallel to the web as at right angles thereto, while figures *A* and *D* indicate that tie plates would materially strengthen the flanges by stiffening them against twisting.

TABULAR RESULTS OF BUREAU OF STANDARD TESTS

Assembled by the Engineering News Record.

TABLE I. TESTS OF SOLID-SECTION COLUMNS; 198 TESTS MADE IN CO-OPERATION WITH A COMMITTEE OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS

NOTE—The slenderness-ratios in the tables are abnormal, as the tests were made with truly squared ends. Divide the tabular slenderness-ratios by 2 to get the pin-end values.

Slenderness Ratio l/r	Nominal Area of Cross-Section, Sq.In.	Ultimate Strength, Lb per Sq.In.	Average
Type 1, Light Type 1a, Heavy Type 1b, Extra Heavy	Plate-and-angle I-section; 1 plate 6 in. wide, 4 angles 5x3 in		
LIGHT, 5/16-IN. METAL			
20	11 475	[37,000] 38,400 [37,000] 38,200 [37,000] 38,000	[37,000] 38,200
50	11 475	32,400 32,800 33,000	32,700
85	11 475	32,100 31,400 30,000	31,200
120	11 475	28,600 27,600 28,700	28,300
155	11 475	27,000 25,600 26,000	26,200
HEAVY, ⅝-IN. METAL			
20	22 19	[29,000] 43,200 [30,000] 43,300 [29,000] 44,300	[29,300] 43,600*
50	22 19	29,000 28,900 29,700	29,200
85	22 19	28,000 28,200 28,000	28,100
120	22 19	25,000 25,500 25,800	25,400
155	22 19	22,200 23,200 22,600	22,700
EXTRA HEAVY, 11/16 AND ¾-IN METAL			
85	28 61	28,200 27,000 28,000	27,700
Type 2, Light Type 2a, Heavy	Box section, 2 6-in channels, 2 8-in plates		
LIGHT, 10½-LB CHANNELS, ¼-IN PLATE			
50	10 18	33,100 34,000 34,500	33,900
85	10 18	33,700 32,300 31,700	32,600
120	10 18	28,300 28,700 30,900	29,300
155	10 18	26,100 27,800 25,600	26,500
HEAVY, 15½-LB CHANNELS, ½-IN. PLATE			
50	17 12	32,200 32,500 32,500	32,300
85	17 12	30,700 30,000 31,000	30,600
120	17 12	28,500 28,800 27,000	28,100
155	17 12	24,700 26,000 25,000	25,200
Type 3, Light Type 3a, Heavy	Box section 2 5-in. channels, 2 9½-in plates		
LIGHT, 6½-LB CHANNELS, ¼-IN PLATE			
50	8 65	33,800 34,300 34,300	34,100
85	8 65	32,500 31,800 33,000	32,400
120	8 65	31,600 30,200 30,000	30,600
HEAVY, 11½-LB. CHANNELS, ½-IN. PLATE			
50	16 26	29,800 29,300 29,500	29,500
85	16 26	28,000 28,000 28,000	28,000
120	16. 26	28,000 26,100 26,600	26,900
Type 4, Light Type 4a, Heavy	I-and-channel section; 1 8-in I-beam, 2 8-in channels.		
LIGHT, 18-LB. I, 11¼-LB. CHANNELS			
50	12 03	35,700 36,500 38,600	36,900
85	12 03	34,000 33,900 34,000	34,000
120	12 03	32,000 30,700 33,100	31,900
155	12 03	24,000 23,200 23,500	23,600
HEAVY, 20½-LB. I, 18¾-LB. CHANNELS			
50	17 05	30,300 29,000 28,000	29,100
85	17 05	26,000 26,400 27,500	26,600
120	17 05	23,500 23,900 24,200	23,900
155	17 05	23,900 23,000 23,000	23,300
Type 5, Light Type 5a, Heavy Type 5b, Extra Heavy	Bethlehem H-section; 8 in. H-beam.		
LIGHT, 32-LB H			
20	9 17	[39,000] 39,600 [40,000] 41,000 [40,000] 41,000	[39,600]* 40,500
50	9 17	38,000 38,000 38,000	38,000
85	9 17	36,000 34,000 33,000	34,300
120	9.17	31,000 33,900 31,000	32,000
HEAVY, 62-LB. H			
20	18 27	[35,000] 35,600 [37,000] 37,800 [36,000] 36,700	[36,000]* 37,000
50	18. 27	34,000 36,100 36,000	35,400
85	18 27	33,900 32,000 31,000	32,300
120	18 27	30,000 30,000 30,000	30,000
EXTRA HEAVY 91-LB. H			
20	26 64	[25,000] 54,800 [26,000] 56,600 [25,000] 54,500	[25,300]* 55,300
50	26 64	27,100 24,000 24,600	25,200
85	26 64	24,800 22,500 23,000	23,400
120	26 64	21,800 21,000 21,000	21,300
Type 6, Light Type 6a, Heavy	Reinforced I-section, 10-in I, 2 11-in flange-plates.		
LIGHT, 25-LB I, 5/16-IN PLATES			
50	14 25	31,200 31,500 32,200	31,600
85	14 25	27,800 29,500 30,100	29,100
120	14 25	26,700 27,700 27,300	27,200
HEAVY, 35-LB. I, ¾-IN PLATES			
50	24 04	31,200 32,800 32,400	32,100
85	24 04	27,200 26,800 26,400	26,800
120	24 04	24,900 24,000 25,500	24,800
Type 7—Bulb-Angle Column Section, 6x5/16-in plate, 4 5-in by 10.1-lb bulb angles.			
LIGHT			
50	13 76	34,300 32,700 33,200	33,400
85	13 76	34,000 30,000 30,700	31,600
120	13 76	28,300 26,900 29,000	28,100
Type 8, Light Type 8a, Heavy	Z-bar column, 4 4-in. Z-bars, 1 7-in plate		
LIGHT, ¼-IN METAL††			
50	11 39	35,900 35,100 36,200	35,700
85	11 39	31,400 32,900 34,000	32,800
120	11 39	29,100 29,400 30,400	29,700
HEAVY, ⅝-IN METAL			
50	26 58	33,000 33,500 32,200	32,900
85	26 58	31,300 25,300† 31,500	29,400
120	26 58	27,500 27,000 27,500	27,300
Type 10, Light Type 10a, Heavy	Built-up box section; 4 angles 2x2-in, 2 plates 9-in.		
LIGHT, ¼-IN METAL			
50	10 89	36,100 35,700 35,600	35,800
85	10 89	32,500 31,900 31,900	32,100
120	10 89	28,400 28,300 28,400	28,400
HEAVY, 5/16-IN METAL			
50	18 71	31,400 32,000 32,100	31,800
85	18 71	[illegible] 28,000 29,000	28,300
120	18 71	27,000 25,900 26,000	26,300

* Test values for slenderness-ratio 20 (or pin-end value 10) represent short-block compression The ultimate strengths include the effect of plastic squeezing in the heavy and extra-heavy sections The yield-point values are, therefore, added in brackets [ ]; they are the values of basal column strength, for comparison with the ultimate strengths of the longer columns

† No explanation for this abnormal figure was discoverable

†† The light Z-bar columns failed by twisting (A in Fig 4), but a strength-reducing effect of this action is not detectable in the results

TABULAR RESULTS OF BUREAU OF STANDARDS TESTS

Assembled by the Engineering News Record.

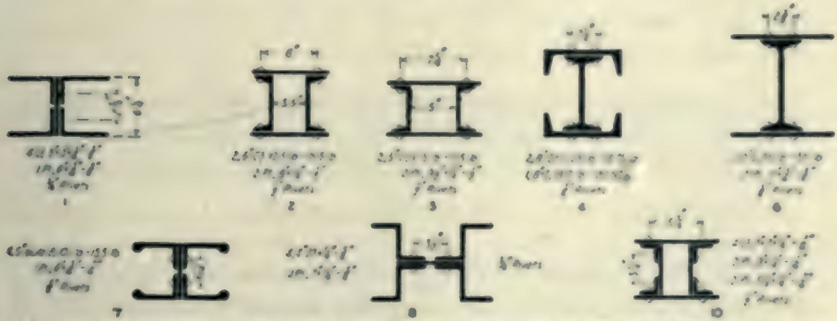

FIG. 2. TESTS OF SOLID-SECTION COLUMNS SHOWED NO INFLUENCE OF SHAPE OF SECTION. TESTED IN LENGTHS OF 50 TO 120 R. WITH SPECIAL TESTS AT LENGTHS 20 AND 155 R.

TABLE II. TESTS OF 8-IN. CHANNEL COLUMNS, LATTICED AND TIE-PLATED IN VARIOUS WAYS. 42 TESTS

Made in Co-operation with a Committee of the American Railway Engineering Association

NOTE.—The slenderness-ratios in the tables are abnormal, being computed as pin-end values, while the tests were made with truly squared ends. Divide the tabular slenderness-ratios by 2 to get the pin-end values.

All light columns, 2 channels 8 in. by 10¼ lb. per ft.
All heavy columns, 2 channels 8 in. by 21¼ lb. per ft.
Flanges turned in, spacing 10¼ in. back to back.

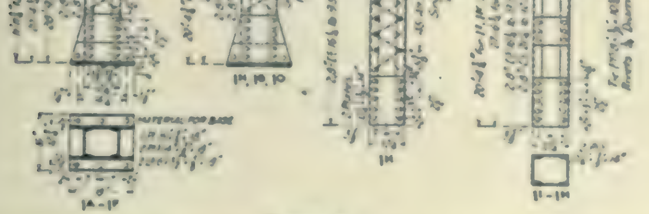

FIG. 3

Tests of latticed and tie-plated columns showed tie-plated columns to be weak—tested in lengths of 50 to 120 r.

LATTICED COLUMNS WITH HEAVY LATTICE

Single Lattice 2½ x ⅜ in., alternate spacing 4½ in., ⅞-in. rivets

Slenderness Ratio l/r	Nominal Area of Cross-Section Sq. In.	Ultimate Strength Lb. per Sq. In.	Average
LIGHT			
1A 50	9.56	37,000 39,100 38,200	38,500
1B 85		32,000 35,000 34,000	34,000
1C 120		31,000 32,000 34,000	33,000
HEAVY			
1D 50	12.50	38,500 37,000 35,000	37,200
1E 85		34,000 34,000 34,000	34,100
1F 120		32,000 29,000 30,900	30,600
LIGHT WITH PLAIN ENDS			
1H 85	9.56	33,300 33,000 33,000	33,900

LATTICED COLUMNS WITH LIGHT LATTICE

Single Lattice 2¼ x ¼ in., alternate spacing 4½ in., ⅞-in. rivets

Slenderness Ratio l/r	Nominal Area of Cross-Section Sq. In.	Ultimate Strength Lb. per Sq. In.	Average
LIGHT			
1N 85	9.56	33,600 33,500 34,200	33,800
1O 120		34,200 32,000 33,000	33,100
HEAVY			
1P 85	12.50	32,200 31,000 32,600	31,900

TIE-PLATED COLUMNS, 8½-IN. TIE-PLATES 10¼ IN. LONG

Three Tests, long spacing, distance between nearest rivets such that l/r represents for one channel equals l/r computed for whole column.

One Test, short spacing, tie-plates spaced about ½ as far as in the preceding.

Slenderness Ratio l/r	Nominal Area of Cross-Section Sq. In.	Ultimate Strength Lb. per Sq. In.	Average
LIGHT			
1J 85	9.56	27,000 25,100 24,000	25,400*
HEAVY			
1L 85	12.50	26,500 24,100 25,000	24,120*
1M 120		15,000 15,300 15,000	15,100*
LIGHT, SHORT SPACING			
1K 85	9.56	27,000 26,000 27,000	26,600*

* The tie-plate spacing represents pin-end slenderness-ratios exceeding those of the entire column by 50 to 100%. In the strongest of these tests 26,900 lb. per [illegible] of 85, the tie-plate spacing gives a ratio in the individual channel of 63. In the lowest 15,100 the column has pin-end ratio 60 while the channel ratio is 120.

For application of these strength values to pin ends the lengths or slenderness ratios should be taken at substantially six tenths and these tests therefore fail in that the range thru slenderness ratios was too limited.

It has long been recognized that the elastic compressive strength of the short column of built up sections is limited by the compression yield point of the metal and the tests just discussed are confirmatory.

Since different lots of steel accepted as meeting the same required tensile strength and tensile yield point are shown by these tests to differ widely in compresive yield points developed they demonstrate that columns supposedly alike may differ widely in strength.

How this unsatisfactory condition may be remedied and metal of high compressive quality may be obtained from the mills under commercial conditions is a matter involving a discussion of alloy treatment in the process of manufacture.

CHAPTER XIX

PHYSICAL PROPERTIES OF STEEL DEPENDENT UPON FURNACE PRACTICE TEMPERATURE OF ROLLING AND CONSIDERATIONS DETERMINING ECONOMIC GRADE OR QUALITY

1. Alloy Treatment in Smelting Structural Steel may be defined as the means taken to lessen the oxygen and oxide content of a melt to or below an allowable minimum and secure sound ingots reasonably free from gas pockets or blowholes and low in the deleterious impurities of phosphorus and sulphur.

Certain elements having stronger affinity for oxygen at steel melting temperatures are added to the molten metal. The oxygen leaves the iron to combine with such added elements forming new products insoluble in the iron which are precipitated and leave the metal. Further, there is a tendency for certain impurities to collect or segregate in an injurious way in the upper central part of an ingot or casting but the addition of certain elements lessens this tendency.

Thus the early product of the Bessemer converter was largely unfit for use as the metal poured from it was full of blowholes. By adding molten spiegeleisen containing twenty percent of manganese fairly sound setting steel was produced. (See Section I.) To insure certainty of the desired results some excess of manganese over and above the minimum required as a curative of these difficulties is added in Open Hearth practice as well as Bessemer. This excess varying from twenty to one hundred points (i.e., hundredths of a percent) does not greatly affect the physical qualities of the softer grades of steel as indicated by ordinary methods of testing. It tends to increase the tensile strength an amount increasing with the carbon content. It apparently enables the specimen better to withstand a cold bend test; BUT IF A REVERSE BEND is superposed upon the positive the effect of higher manganese in reducing the dependability of the material under compression is manifest. The tensile and compressive resistance of simple carbon steels are approximately in balance. Manganese increases the tensile elongation and reduces the elastic compressibility of the material to such a marked extent that a fracture in a bend test with high manganese content starts not on the tensile side of the specimen as in carbon steel low in manganese but on the compression side of the specimen.

In cold bending with a hammer the specimen is seemingly tough as long as the hammering bends it the same way but a light blow in the reverse direction may snap the specimen as though it were glass.

To one conversant with these characteristics of excess manganese the deficiency of the Standard Specifications of the American Society of Testing Materials for steel for bridges, ships, cars and buildings is surprising. A quantitative (ladle) analysis for manganese is required but no limit whatever is stated or placed upon the quantity of this element that may be present as an undigested curative in the finished product.

Because manganese plays such an important part in the Bessemer practice and the process is so rapid that it is not under as good control as is the Open Hearth excess manganese is more often found in the Bessemer steel. Concerning the unreliability of Bessemer steel, A. E. Hunt, Journal of the Iron and Steel Institute, Vol. II, 1890, wrote as follows: "Various cases have come under our observation of angles and plates which broke off short in the punching but altho makers of Bessemer claim that this is just as likely to occur in Open Hearth we have as yet never seen an instance of failure of this kind in Open Hearth steel"

The use of good stock determines to a great extent the uniform nature of the product. But it does not influence the solidity of the ingot castings. This depends on the composition of the slag and metal before tapping and on the quantity and nature of the recarbonizing additions.

Campbell compares the efficiency of the elements of manganese, silicon and aluminum in producing soundness on the basis of their power to unite with oxygen, the action being as follows:

3.44 parts of manganese unite with one part of oxygen.
3.44 parts of aluminum unite with 3.01 parts of oxygen.
3.44 parts of silicon unite with 3.93 parts of oxygen.

Hence aluminum is three times and silicon four times as efficient as manganese weight for weight, while they possess an additional value because of their greater affinity for oxygen enabling them to sieze the last traces from the iron and render the melt so much the cleaner.

Added to greater affinity for oxygen aluminum and silicon possess a greater capacity to dissolve or occlude gases which as far as the soundness is concerned is equivalent to destroying them.

Aluminum excess should preferably not exceed ten points. Silicon increases the toughness, the compressive yield point and the tensile strength and for a moderate excess .40 to .50, does not render rolling more difficult.

Reduction of harmful amount of oxygen and oxides in the melt before pouring the ingot involves improvement in quality other than a soundness or solidity of the ingot. Rolling and working the metal improves the finished product by welding more or less perfectly the surfaces of small pockets and reducing their cross sections but such working cannot rectify the quality of metal containing a high oxide content.

A practical grasp of this phase of the problem may best be acquired by the student by over heating sample bars of steel in the forge—burning them as the smith would call it; then subjecting the specimen to the usual cold bend and tensile tests and comparing the results with that secured in tests of undamaged metal.

The rejection of a carload of U-shaped coupler pockets some years ago by the master mechanic of a railroad company resulted in the request to inspect and report upon them for the manufacturer.
Placing several upon the ground the writer struck the first one a sharp blow with a heavy sledge knocking the bottom out of the *U* and fracturing eleven square inches of steel at a single blow in so doing. The others were all broken in a similar manner requiring a second and in a few cases a third blow. The metal of the shank was found good standing a cold bend test of 180 degrees to half the thickness of the stock. Defective shop work, metal burned in forging the bend was the finding and substance of the report. The deficiency of burned (oxidized) metal both in strength and resistance to shock is of like character whether acquired in the Bessemer converter, the open hearth furnace or the forge.

2. Influence of Silicon on Medium and High Carbon Steel was regarded as injurious in early Bessemer practice. Unless specially added silicon rarely exceeds ten points in carefully manufactured mild steel. Its occasional presence in much larger proportion in Bessemer steels indicates* that the steel was cast at too high a temperature reached so rapidly that the carbon was oxidized before the silicon. The unreliability of this metal has been credited to the silicon instead of to careless manipulation in manufacture.

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*Harbard and Hall, Metallurgy of Steel, page 368-9.

For the severe service of locomotive tires twenty to forty points silicon in open hearth steel with fifty to sixty of carbon has been found to give most satisfactory results.

C. P. Sandberg finds the wearing qualities of steel rails greatly improved by the presence of .35 silicon. Campbell quotes the statement of a French steel master that he could only meet the requirements of a particularly severe specification by making rails containing .30 to .40 percent of silicon since a less proportion would not stand the drop tests. Quoting further:

> "Tool steel is subject to the most severe of all tests in the exposure of a hardened edge to the blows of a hammer or shocks of a planer. It was not the laboratory but the requirements of general practice from which was unconsciously evolved the formula for such metal, requiring low phosphorus, low sulphur and low manganese. In this process of natural selection . . . . a large part of the best steel has regularly contained year after year .20 to .80 percent of this element."

The writer has found that basic open hearth steel, low in manganese, phosphorus and sulphur, having an ultimate strength of from 115,000 to 120,000 lbs. per square inch with .30 to .40 silicon in deformed rounds would bend cold 180° and back about a pin four diameters for three quarter bars and three diameters for half inch bars without sign of fracture while steel of equal strength, of lower carbon but with .60 to .80 manganese will stand little reverse bending after bending 100 degrees in one direction.

Silicon increases the tensile strength of the steel about 200 lbs. per point, raises the elastic limit a moderate amount and increases its toughness. With silicon .20 to .24 percent and carbon .16 to .18 heating to a welding heat and quenching in cold water in tests by Hadfield produced little hardening such as occurs in carbon steel and no material reduction in ductility.

Increase of elastic limit and toughness caused by .20 to .40 silicon with .40 to .50 carbon may apparently be attributed to rendering the metal less sensitive to the surface quenching of the water necessary to cool the rolls, thereby maintaining a more uniform structure in the steel in lieu of a very hard exterior and a softer interior of the purer carbon steel and its resultant non-uniformity of resistance.

3. Piping is produced by the shrinkage of the metal in cooling.

The bottom and sides of the ingot cool first and as the metal contracts at the base it settles from the more fluid top forming a conical cavity or pipe. Although the addition of aluminum and silicon render the steel sound they increase the amount of pipe and the percent of discard or length sheared off necessary to secure solid billets.

Sir Robert Hadfield, Iron and Steel Institute Journal 1912 describes a method of producing ingots free from pipe giving satisfactory results in his own works. It consists of a feeding head the sides of which are rammed up with sand placed on top of the ingot mold. The ingot is poured and the head filled to within four inches of the top and a 1½ inch layer of ground cupola slag is placed on the top of the molten metal covered by a layer of charcoal which is kept at a high heat by a blast of air. The slag by fusion forms a neutral layer preventing loss of heat by radiation and absorption of the carbon of the charcoal by the steel. The heat developed by the combustion of the charcoal maintains the top surface of the steel in such fluid condition during the cooling of the base of the ingot that the pipe or cavity that would otherwise form is kept filled with molten metal in the ingot proper thus confining the cavity to the feeding head only.

4. **Segregation** is the term applied to the concentration of impurities in the cooling of large ingots. These impurities such as phosphides, sulphides, carbides, etc., dissolved in the iron differ from iron in that their temperature of solidification is much lower than that of iron. Having a lower point of solidification than iron they tend to separate from it as the outer surface of the ingot hardens concentrating toward the more fluid portion, i.e., the center as the ingot cools from the outside. Being lighter than iron they tend to rise toward the top enabling their removal by shearing off and discarding the top portion.

The amount of this discard was arbitrarily fixed at thirty percent in the specification for shell steel in the World War, but in steel for railroad bridges its amount is left solely to the tonnage or quality proclivities of the rolling mill superintendent in our A.S.T.M. and A.R.E.A. specifications.

5. **Ladle and Check Analyses;** are provided for under the above specifications. The ladel analysis discloses the average chemical make up of the melt at the time of pouring the ingot in points of carbon, manganese, sulphur and phosphorus and silicon

(when specified). The specified check analysis in general provides that analysis of the finished steel shall not exceed the tolerance for phosphorus and sulphur of the ladle analysis by more than 25 percent. To find that portion of the finished product rolled from the central top portion of the ingot containing the greatest segregation is the question upon which the effective utility of this limitation depends. If the sample be required to be taken from the central top of the ingot along the plane of discard its utility would be unquestioned as a limit of tolerance of segregation.

In the Journal of Iron and Steel Institute 1905, page 204 analyses are given by Talbot of ingots of acid and basic open hearth steel with and without the addition of four ounces of aluminum from which the following table is copied. The borings were taken from the top to the bottom at six inch intervals, the ingots measuring 18 inches by twenty two by sixty seven inches. The average of all drillings for the ingot without aluminum was S. .059, P. .055, C. .41 and the average from the ingot to which aluminum had been added was S. .060, P. .052, C. .39. Without aluminum excessive segregation appears down the central portion for half its height; sulphur segregating most, phosphorus next, then carbon.

ANALYSIS TAKEN FROM CENTER AND OUTSIDE OF TWO INGOTS, FROM TOP TO BOTTOM AT DISTANCES OF 6 INCHES APART

	Without Aluminum						With Aluminum 4 oz. per ton					
No.	Center			Outside			Center			Outside		
	Sulphur	Phosphorus	Carbon	Sulphur-	Phosphorus	Carbon	Sulphur	Phosphosus	Carbon	Sulphur	Phosphorus	Carbon
Top 1	.053	.051	.41	.035	.047	.33	.038	.035	.32	.067	.054	.39
2	.049	.052	.39	.031	.034	.32	.042	.040	.35	.066	.057	.39
3	.267	.197	.95	.018	.030	.31	.078	.061	.48	.065	.056	.39
4	.141	.109	.59	.027	.036	.32	.086	.063	.43	.066	.055	.40
5	.097	.083	.47	.034	.040	.35	.081	.049	.34	.068	.067	.41
6	.081	.065	.46	.036	.046	.37	.059	.054	.38	.066	.058	.42
7	.056	.061	.46	.040	.048	.38	.067	.052	.38	.064	.057	.41
8	.056	.053	.38	.041	.046	.38	.057	.049	.36	.058	.052	.41
9	.043	.045	.35	.039	.044	.36	.053	.049	.36	.058	.056	.40
10	.048	.047	.35	.044	.046	.38	.056	.050	.36	.056	.056	.38
11	.047	.045	.34	.048	.048	.41	.057	.047	.37	.060	.053	.38
12 Btm.	.051	.050	.39	.053	.054	.43	.056	.049	.38	.065	.054	.39

The uniformity of the distribution of sulphur carbon and phosphorus effected by the small amount of aluminum is remarkable in view of the wide variation in the untreated ingot. The high carbon, sulphur and phosphorus content in the third drilling exceeding a reasonable tolerance for sulphur and phosphorus by four hundred percent indicates undesirable physical properties and lack of dependability such as some years ago the writer found in some 6 × 6 × ¾ angles for the shoe of a railroad bridge. A sharp blow of an ordinary machinists hammer was all that was requisite to break a chunk out of the three quarter inch thick leg.

6. **Rising Steel** is the term applied by the steel master to mild steel poured into the ingot mould while still boiling instead of "killing" or quieting the melt by alloy treatment. Segregation and pipe are reduced by this proceedure but the resulting ingot is spongy and dependence is placed upon the weldability of mild steel during rolling to remedy the defect of unsoundness of the ingot.

For the tension member flattening out of the gas pockets and welding their sides more or less imperfectly is not so objectionable since the pull tends to press the welded surfaces together; conversely when the metal is opposing resistance to compression these surfaces are subjected to a strain tending to pull them apart and the reduced resistance of the metal compared to wrought iron seems otherwise unaccountable.

By the foregoing practice an increased tonnage output of six to eight percent is attained permitting a reduced cost to the purchaser of perhaps two percent. Viewed from the economic standpoint, this is saving the dime at a cost of the dollar when the structural steel of today fails to develop the strength of the wrought iron of thirty years ago in tests of full sized compression members.

The "bushel" steel bar of the faggot mill bends beautifully but with lead-like resistance because of the imperfect longitudinal welding of its filaments therein differing from real steel. A dynamometer attached to the bending machine in making the bend test would bring this divergence of resistance out in a striking manner and the same would be true if applied to bars rolled from the rising steel ingot when contrasted with those produced from the sound ingot. The reverse bend test would tend to bring out its relative lack of resistance to compression. As the size of the ingot increases the segregation and pipe increase and it may be questioned if in doing things in a big way our mills have not carried the size of ingots to excess at the expense of quality of product.

7. **Critical Temperatures:** On heating up a piece of steel and

allowing it to slowly cool there are certain points at which instead of the cooling proceeding normally, heat is evolved. These points are known as the critical points of which a complete record is now available ranging from the softest iron to the hardest steel. On heating the sample the reverse phenomena to those observed during the cooling occur. As might be expected the time element enters into the phenomena.

For physical equilibrium the critical points of heating and cooling should be the same but because the heating and cooling of the outside of the specimen differs from the condition on the interior the apparent temperatures of the critical points during heating and cooling differ. Those observed during heating up are designated *Ac*, while those observed during cooling are designated as *Ar*. In order to induce the retardation *Ar* or rather the change that such retardation implies the steel must first be heated past the point *Ac* and reciprocally the change which occurs at *Ac* cannot take place unless the steel has first cooled to a point below *Ar*. In pure iron there are two critical points during the cooling Ar_2 and Ar_3 while in low carbon steels there are at least three well recognized critical points Ar_3, Ar_2 and Ar_1.

Harbord and Hall give the approximate temperatures at which the maximum heat evolution takes place during the cooling of steel of various carbon content at the critical stages as follows:

	Ar_{mc}.	Ar_3.	Ar_2.	Ar_1.
Pure Iron	...	900	762	...
0.12 percent C	...	850	762	715
0.24 percent C	...	820	762	715
0.53 percent C	...	730		715
0.89 percent C	...	715		
1.31 percent C	883	715		

Ar_1 of the table depends on the carbon present and it does not appear in pure iron. The condition of thec arbon above Ar_1 is in the state of hardening carbon while below this point it is in the state of cement carbon or carbon which is combined with iron in the form of definite carbide, Fe_3C.

Interesting but* divergent theories are presented by metallurgists to account for the change in structure which occurs at these critical temperatures. Thus we have the old theory in which carbon is considered to be dissolved in the molten iron and the variation in the micro structure of the steel arises in a manner analogous to crystallization in solidifying solutions.

*Metallurgy of Steel, Harbord and Hall, Physico Chemical properties of Steel by Edwards, etc.

The "Allotropic" theory is explained by Austen as a change of internal energy occuring in an element at a critical temperature not necessarily accompanied by a change of state. Under this theory Osmond considers that hardened steel owes its properties to the presence of "Beta" iron which is hard and brittle by itself at ordinary temperature. Iron below the point Ar_2 possessing the molecular form "Alpha" or soft form but above the critical point the alotropic form or Beta iron exists. Carbon maintains iron in the Beta Condition during slow cooling in inverse proportion to the amount of carbon in the metal.

The carbon and subcarbide theory maintains that the phenomena observed in the hardening of steel is explained by changes in the condition of the carbon. That carbon can exist in two states in steel; first as hardening carbon at a temperature above the critical points which when the cooling takes place rapidly, as by quenching, is retained in the same state. Slow cooling on the other hand from above the critical points causes the hardening carbon to pass into the form known as cement carbon, and combined with the iron to form the carbide Fe_3C.

Stress Theories have been advanced to account for hardening on the ground of a severely strained condition resulting from the shrinkage of the outer layer during quenching with pronounced crystal twinning.

This field of theoretical investigation and speculation appeals to the engineer only as it is related to the physical properties of the material which he is using. He learns by experience that under cold working the yield point of iron or soft steel is raised, that its ductility lessens, its density is slightly decreased, it rusts more readily, becomes less resiliant and harder in the sense that it requires a greater pressure to produce flow, and that its position in the thermo-electric series changes along different axes.

As ordinary iron or soft steel is given a set by cold working, the displacement of the material is accompanied by the evolution of heat. Conversely when the effect of this displacement or internal strain is relieved by heating when it arrives at that critical temperature where the change takes place rapidly an absorption of heat occurs. In other words, the effect of over strain may in a large measure be eliminated by heating above the lower critical temperature and allowing the metal to cool slowly since the energy expended in performing the work on the metal is restored to it by the absorption of energy at the critical temperature in heating.

The texture or grain becomes coarse if it is heated far above A_2 and below A_3 and slowly cooled unless the metal is subjected to the mechanical work of hammering or rolling. In like manner the fineness of the structure or grain of the metal is a maximum when this work is continued down to just above the lower critical point.

It is found in respect to rails that where the last pass is held back until the temperature has fallen to between 1600 and 1500° Fahr., instead of finishing them at or about 1800° that a much finer grain steel is the result which wears well and gives the best service. Bars which are finished at the lower temperature are stronger and stand a more severe bend test than those which are finished at higher heat.

Under present mill practice of paying for rolling by the ton, the color (temperature) of the metal at the final pass of the rolls is given scant consideration and the uniform quality of the product suffers accordingly. The wide gap in relative bending resistance of the thick and thin specimen would in a large measure disappear were there coupled with the tonnage price a bonus for uniformity and a penalty for non-uniformity of the output. Then the color of the metal would be given consideration in connection with the tonnage output with an improvement in the quality four times the differential which would satisfy the mill men for exercising the care necessary to produce the desired result.

The spoken appreciation of quality (provided the manufacturer can deliver it at the lowest pound price on the commercial market) is an ineffective consideration, non-productive of economic results as illustrated by the continued manufacture of structural steel inferior for compression members to the wrought iron of years ago.

The move on the part of structural steel fabricators to increase the present working stresses permissible under building code rules for structural steel is warranted only to the extent that it is accompanied by improvement in product, under specifications designed to secure steel possessing higher resistance rather than that which can be produced at the lowest unit pound price.

The civil engineer has lagged far behind the mechanical engineer in taking advantage of the increased knowledge and facilities for the production of high grade steel. The automotive engineer is securing great increase in strength and toughness with reduced weight and cost to the user and the civil engineer may profit by his example.

8. Alloy Steels are distinguished from alloy treated steel by the presence of the alloying element in sufficient quantity to radically change the properties of the simple steel increasing the elastic strength or hardness and toughness of the metal many fold thereby advantageously fitting it for special service when properly heat treated.

Alloy steels, like the simple steel, are made in acid or Basic, Open Hearth furnaces, in the electric furnace or by the crucible process. The most inexpensive method being empioyed that yields a product satisfactory for the purpose in view.

As used in automobiles, alloy steels are considered with respect to their increased strength economically advantageous only when heat treated. Ordinary nickel steel having an analysis of C.25, *Mn.* 60, *Ni.* 3.5 untreated will have an ultimate strength approximately 95,000 pounds per square inch, elastic limit of 55,000 lbs. and an elongation of 20 percent in eight inches; but when quenched in water at 800 C. with hardness drawn in air at 316 C, the tensile strength runs over 200,000 lbs. per square inch and the elastic limit around 170,000 pounds per square inch or if the hardness were drawn in air ar 538° C a tensile strength of approximately 140,000 pounds per square inch and an elastic limit of 120,000 lbs. per square inch might be expected with elongation equal to that of the untreated specimen.

Thus the same steel may be given a wide range of properties and any desired set of properties within this range may be obtained solely by varying the heat treatment. The principal variant is the degree of the second heating. The lower this is the stronger and stiffer the steel and the higher the weaker and more ductile it becomes.

Successful heat treatment requires uniform heating and uniform cooling and quenching and after quenching the piece be made true to form as quenching, tho carefully done, tends to leave the piece treated warped or twisted. The simplest solid or relatively thick forms are those most readily treated such as thick armor plate in contradistinction to the thin structural plates, etc. The trouble in straightening the plate or shape after heat treatment lies in the springiness of the metal after treatment rendering it difficult to give it that precise set necessary to counteract the bend or twist that has been induced in the quenching process. Practical means for accomplishing this result undoubtedly involves pressing the heated material during the quenching process, requiring expensive equipment but no insuperable obstacles to its realization.

Carbon steel eye bars are now being treated by the American

Bridge Company and some work has been done in treating nickel steel bars. A working stress in tension two and one half times that permissible for medium steel may be safely adopted in heat treated material developing a yield point value of 120,000 to 130,000 pounds per square inch and possessing the degree of toughness and ductility indicated by an elongation equal to that of intermediate grade steel.

The writer does not consider it advisable to use a working stress as high as 45,000 pounds with a yield point no higher than 75,000 pounds as has been recently proposed for heat treated carbon steel bars for such relatively high stress does not give the customary margin between the yield point and the working stress to provide safely for the range of imperfect elasticity known to exist between the thermal limit of proportionality and the apparent limit of elasticity of shape.

The elements which are commonly used in alloy steels are manganese, nickel, chronium, silicon, vanadium, molybdeum, cobalt, tungsten and others, and various combinations of these elements. The number of possible alloys is almost unlimited and while the number of useful alloys are many but a fraction of this number are commercially so because the same useful properties and characteristics of one alloy may be duplicated in another and different alloy at less cost.

In the field of bridge construction ordinary nickel steel with 3.24 to 3.50 precent nickel and a natural chrome nickel steel made from ore mined at Mayari, Cuba, carrying enough nickel to give 1.3 to 1.5 nickel and 2.50 to 3.00 percent chronium in crude iron smelted therefrom.

In the conversion of this iron into steel little nickel is lost but the chronium content is largely reduced by oxidation. Alloy steels for riveted bridge members have been customarily used in their natural or untreated state involving a divergence of opinion between the automotive engineer and the bridge designer in regard to the enonomy of the strength secured by their use.

The Journal of the Iron and Steel Institute of 1889 contains a paper by Mr. James Riley, furnishing one of the earliest disclosures of the effect of nickel alloyed with steel. Since then, experiments on heat treated armor plate have made the advantages of nickel steel widely known and given rise to an exaggerated estimate of its utility when untreated.

The physical qualities of nickel and carbon steel of similar strength manufactured in an acid open hearth furnace of the Pennsylvania Steel Company, are compared by Campbell in the following table:

THE PHYSICAL QUALITIES OF NICKEL STEEL AS COMPARED WITH CARBON STEEL OF SIMILAR TENSILE STRENGTH

(Note:—All steels were made in an acid open-hearth furnace by The Pennsylvania Steel Co.)

Kind of Steel	Composition per cent				
	C.	Mn.	P.	S.	Ni.
Nickel	.24	0.78	.032	.027	3.25
Hard forging	.30 to .35	.60 to 1.00	.03 to .05	.03 to .05	nil.
Forging	.25 to .30	.60 to .80	.03 to .06	.03 to .07	nil.

Shape of Member	Kind of Steel	Ultimate Strength Lbs. per Sq. In.	Elastic Limit Lbs. per Sq. In.	Elastic Ratio percent	Elongation in 8 ins. percent	Elongation in 2 ins. percent	Reduction of area percent
Rounds	Nickel	86015	63575	73.9	20.19	34.00	46.3
	Hard forging	87663	58055	66.2	16.70	24.44	30.3
	Forging	78066	51793	66.3	23.94	....	52.0
Angles	Nickel	86960	58553	67.3	21.75	39.67	50.5
	Hard forging	87820	54153	61.7	19.25	34.83	43.3
	Forging	76970	49544	64.4	....	....	49.6
Universal Plates Longitudinal	Nickel	85773	58410	68.1	21.08	39.25	52.0
	Hard forging	82773	50163	60.6	20.50	37.67	47.0
	Forging	78996	46654	59.1	26.78	....	52.1

The effect of the nickel is to increase the ultimate strength, elastic limit, elongation, and reduction of area somewhat with a given percentage of carbon but the difference is not great and the increase in cost of 3.25 to 3.50 percent nickel is approximately forty dollars per ton of the steel produced.

Whether such moderate increase in strength and toughness may not be secured at a lower cost by alloy treatment or with other and different elements is the practical question from the commercial standpoint. Nickel alloy is more difficult to roll than carbon steel. To off set this higher manganese content than is desirable in carbon steel is used to increase the hot ductility of the metal. Bessemer rail makers in the early introduction of steel looked upon manganese as a cure-all for bad practice, the steel often containing from 1.50 to 2.00 percent. Later, as it was found that rails with high manganese content were brittle in service the permissible maximum as been gradually lowered. In harder steels, experience taught the same lesson. In the structural shape manganese should be kept lower than the limit found permissible in rails to avoid the treacherous characteristics which develop from its effect in reducing the yield point in compression as indicated by the reverse bend test.

Chrome like manganese is small amounts, increases the hot ductility of the metal, raises the elastic ratio and tends to render the hot surface of the metal less sensitive to quenching during rolling, permitting the use of a higher carbon content with safety in structural metal which is to be used untreated. A limitation of manganese content to .35 with carbon not exceeding .45, silicon not over .40 and chrome .15 and .20 should secure an untreated steel of higher compressive resistance than structural nickel steel of our standard specifications with the same carbon content but higher manganese. It would undoubtedly present greater toughness in a reverse bend test.

The cost of such silicon steel would under run that of nickel steel since the silicon would cost little more than the manganese which it displaces and the chrome would not add a dollar a ton to the cost of the metal instead of 35 to 40 for the nickel.

For reinforced concrete construction a higher quality of steel would cost less for the same strength and impart a greater degree of security and dependability to the work since the stress line analysis of concrete beams of Section II shows that the concrete is not stretched in parallel with the steel and that its cracking is caused by shear distortion, a fifty percent increase in the ordinary working stresses of the reinforcement becomes permissible with a stronger dependable grade of steel.

Silicon steel with an ultimate strength of 95,000 to 120,000 pounds per square inch with a yield point above 60,000 can be secured on the market today for from five to eight dollars per ton above the cost of ordinary reinforcement. There is therefore an opportunity to save twenty percent of the cost of the steel and with high grade material and a proper design of the reinforcing bars if deformed. The present average of from three to five percent of defective cold bends would be eliminated.

This estimate of the average number is low because even with structural grade steel there are many deformed bars which get pinched and cracked in bending and with higher carbon steel excess manganese, phosphorus and other impurities result in partial fracture in cold bending.

Deformations which relieve the pressure of the roll upon the bar across the top or bottom induce planes of weakness thru the section which should be avoided. Cross ribs or projections should be squeezed out on the quarter of the bar so that the top and bottom pressure will be maintained thruout the pass. Excessive deformation reduces efficiency as a tension member which constitutes the primary function of the steel in the combination.

With a good concrete, plain round bars have never failed because of deficient bond. Slight roughing and moderate deformation may permit the elimination of the hooks but it should not be carried to the extent of weakening the section in resisting the primary stress for which it is designed.

When our building code laws take cognizance of the quality of the metal which may be economically and scientifically used as reinforcing means the uncertainty involved in damaged cold bends will be largely eliminated and the safety of the structure will not depend on the favorable circumstance that the greatest tension does not generally occur where the bars are bent up and that as the concrete hardens it develops a capacity to resist shear which the designer does not credit to it in his computations. Obviously high grade steel will never be furnished under a code that permits no increase in working stress for improved strength, toughness and dependability of the material.

The idea of a higher and better quality of steel for concrete steel construction cannot be expected to prove popular with those manufacturers who now regard this field of construction as the happy dumping ground for all kinds of off grade stock.

CHAPTER XX

TESTS OF LARGE BRIDGE COLUMNS APPLIED TO THE DETERMINATION OF ECONOMIC PRACTICE

1. The Strength of a Column Increases with the Hardness of the Metal and depends more largely on the carbon content than upon other considerations. To demonstrate this statement we have the tests of Mr. Waddell recorded in Trans. Am. Soc. C. E., Vol. LXIII, June 1909, page 250, thirty foot struts, nickel steel and carbon steel.

The steel analysis of Waddell's test columns gave .19 to .23 carbon, manganese .45 to .56. His nickel steel contained from 3½ to 4 percent nickel, carbon .36 to .46, manganese .62 to .76. The ultimate strength of the nickel steel had a maximum of 112,000 pounds per square inch, an average over 90,000. The ultimate strength of the carbon steel was 60,000 minimum to 66,000 maximum. By doubling the carbon Mr. Waddell secured in his untreated nickel alloy steel a third greater ultimate strength and double the load producing a set of .015.

Strut No.	Nickel Steel			Carbon Steel				
	Load per square inch Producing			Load per square inch Producing			Ratio: Nickel Steel / Carbon Steel	
	Set = 0.005 in.	Set = 0.015 in.	Failure	Set = 0.005 in.	Set = 0.015 in.	Failure	Set = 0.005 in.	Failure
1	39 800	41 600	44 400	....	....	29 600	....	....
2	42 500	....	47 200	16 600	22 200	32 400	....	....
3	....	....	42 500	17 000	20 400	29 600	....	....
Average	41 200	....	44 700	16 800	21 300	30 500	245	147

We will now compare these results with the tests of Mr. Dagron which with manganese .79 to .83, and carbon of .26 to .27, or a third lower carbon than the nickel steel developed on an eight inch lattice channel column 24 feet long, an ultimate strength over 40,000. It appears therefore that as the failure of the nickel steel columns occurred at only 2,000 to 4,000 pounds higher ultimate stress than the carbon steel with but two-thirds the carbon content that the nickel played a negligible part in increasing the strength of the member.

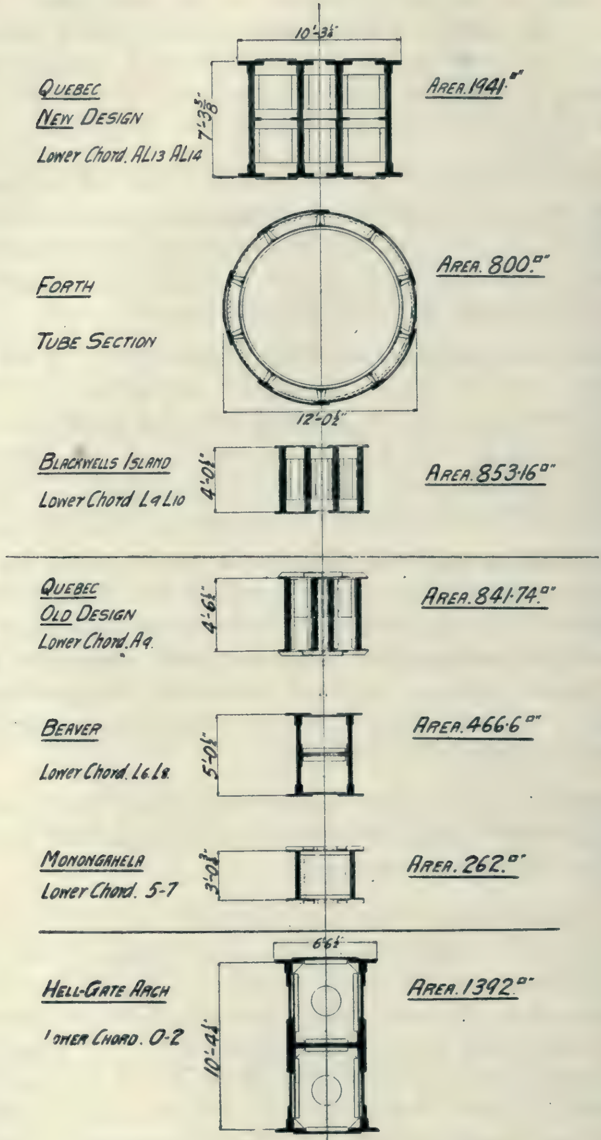
The columns differed thus: In Dagron's tests the pin was parallel to the web; in Waddell's tests the pin was normal to the web, so that the strain of the lacing would be less in producing buckling of the flange in the Waddell tests than in those of Mr. Dagron. The failure of both sets was by crippling of the flanges.

The increased hardness caused by higher carbon in the nickel steel would result in a more favorable comparison with shorter lengths. Waddell's test column sections were web plates 12 × 3/8″, four flange angles 3× 3 × 3/8″, lacing bars 2½ × 3/8″, rivets 7/8″. Mr. Dagron's columns were made up of two plates 8 × 1/4″, four angles 2¼″ × 2¼″ and lacing 1¾ × ¼ with 5/8″ rivets.

2. The Strength of the Column Is Increased as the distribution of the metal further from the axis of the column increases, because the failure of the column is caused by the unbalanced shear distortion which increases the stress over and above the average stress an amount increasing as the distance from the gravity axis of the section, or for laced channels the more the metal is concentrated in the flange the greater the strength.

A comparison of large chord sections reproduced from the Transactions of the Engineering Institute of Canada on the following page may serve to illustrate this relation.

The strongest section there illustrated for the area is obviously that of the Forth Bridge, tube section. The weakest is the Quebec old design lower chord A9, of but 5/8ths the strength computed by ordinary formulas. The total flange area was about four percent of the section contrasted to thirty to forty percent in the flanges of the ordinary laced channel column. The chord of the Hell Gate arch has a substantial cover plate at the top and bottom and ranks next. The Quebec new design has relatively a smaller proportion of metal in the flanges than the channel and Z-sections of which tests have been tabulated. Its width of flange is relatively smaller than that of the rolled channel. But the arrangement of four webs laced with a very small slenderness ratio in one direction enabled the section to develop a close approximation to the yield point value of the individual



— Diagram III —
Sections of
— Compression Chords. —

plates and angles in the 1913 tests published page 211 Rept. Government Board of Engineers Quebec Bridge, Volume 1.

3. **Tests of Five Large Steel Bridge Columns** are described in Trans. Am. Soc. C. E. LXXII, page 429 by James E. Howard. The members were 36 feet 4 inches in length made up of two webs 30 × 11/16ths and four angles 6 × 6 × 5/8ths". The angles were turned in and laced with $2\frac{1}{2}$ × 3/8" bars 7" alternate spacing. The webs were 20-5/8" center to center. Four cross diaphrams of 7/16" plate, four angles $3\frac{1}{2}$ × $3\frac{1}{2}$ × 7/16 × 2'4" divided the column length into five parts. The ratio of L/R is 47, sectional area approximately 90 inches. Loads applied ranged from 29,000 to 30,500 pounds per square inch. The steel varied in carbon from .18 to .27 points, manganese .47 to .54 with low phosphorous and sulphur content. The elastic limit of the plates and angles varied from 33,000 to 38,000; ultimate tensile strength from 50,000 to 53,000 pounds per square inch.

The columns tested satisfactorily to nearly the elastic limit of the material when one failed suddenly. The tests of the others were not brought to a conclusion. The pins were normal to the web and in some of the columns detail weakness in pin bearing developed.

The important lesson which this series of tests teaches and which may well be studied by the student is that of the divergent character of failure of mild steel in the compression member and in the eye bar. In the tests of an eye bar this grade of steel will stretch out gradually without necking down or approaching final failure in a twenty foot bar until the material has been elongated fifteen or even twenty percent of its length. In the column on the other hand it may collapse almost without warning. The soft material does not exhibit toughness under compression because, if it flows readily, instead of accumulating set slowly, the failure will be sudden. On the other hand, if it is hard enough to deform slowly and receive but relatively slight set as the yield point is passed, it becomes for that reason a safer, tougher and more dependable member.

4. **Tests of Scale Models of Large Compression Members** are resorted to by the engineer to determine the probable strength of the members which are beyond the capacity of present day testing machines. Such a series of tests is presented in the studies of the Bureau of Standards of large compression members, Technological Paper No. 101.

Structural and mechanical data concerning the columns tested are shown in Table No. 1, and the relation of the effective elastic limit of the column and given yield point of the steel and ultimate strength are shown in the diagram in which the character of the metal is stated at the top of the column, the name of the column corresponding to that in Table 1 at the bottom.

Comparing the relation of the effective elastic limit of the column with the yield point value of the material it is to be noted that even for the small slenderness ratio of 15 and 16 the effective elastic limit is less than half the yield point of the material of which the sections were made up in the case of four webs laced of Memphis Bridge sections.

Investigating these built channels the ratio of the flange area to the web area in Mayari steel specimen LC_0 LC_1 is better than that in LC L_0C_1 carbon steel and a relatively higher elastic limit and ultimate strength is the result.

Considering the whole series the use of 1/2 inch rivets in sections which in normal practice would be riveted up with 7/8 inch rivets is an element of weakness illustrated in a picturesque manner by the photograph showing the typical failure of the carbon and Mayari steel with the four webs laced top and bottom. Shearing of the small rivets between the webs and the angles and their failure through flexure or slippage to bind the parts together as 7/8 inch rivets would have done, undoubtedly played a part in reducing the effective elastic limit to the low values shown in the diagram.

The sections marked with the star had forked ends which bent over resulting in reduced strength under the maximum load. (Fig. 59 of the Bureau report).

Fig. 53, Column 31 HC_1 carbon steel illustrates the typical failure of the laced H sections. It will be observed that the lacing consists of diagonal straps with transverse angles. Under the principle of rigidities because the solid web is more rigid than the lattice work of angles and straps the web would furnish the predominant resistance to bending in its own plane and the resistance to twisting would reside in the transverse angles leaving the lattice straps to function only to the extent that they take a certain amount of longitudnnal compression buckling up under it and inducing in the flange buckling tendencies which offset their resistance to longitudinal compression to such an extent that had the columns been built without the strap lacing using the transverse angles only a higher elastic limit and compressive strength might have been secured.

ANALYSIS AND TEST RESULTS

TABLE 2—PROPERTIES OF STEEL

Name of Column	Percent of Elements in Chemical Tests—Averages								Tensile tests, pounds per square inch			
									Yield Point			Break Load Mean
	Ni	C	P	Mn	S	Si	Cr	Cu	Mean	High	Low	
Municipal Bridge												
$U_1 S_1$..	3.51	.29	014	.59	.031	....	...	....	55 500	63 100	50 500	93 300
$U_1 U_1$..	3 59	32	015	59	.033	.	...	...	55 200	63 100	52 040	93 060
Metropolis Bridge												
HC ...	....	.36	012	43	030	....	...	...	40 700	50 000	037500	76 660
HC1.		.36	012	40	030		..	...	44 100	50 000	37 650	75 930
MY....	1.12	.34	.010	.68	.037	.10	.53	10	59 900	68 380	55 890	100 610
MY1. .	1.12	.34	010	68	037	.10	.53	10	62 110	68 380	56 080	01 170
31HC.	...	.28	017	.64	031		.	.	42 900	45 440	37 280	73 400
31HC1	....	.28	.017	64	.031	....	..	...	42 700	45 440	37 280	74 440
32S ...	....	.35	.017	83	.039	.38	....	...	57 200	58 840	51 940	87 700
32S1...	....	.35	017	.83	.039	.38	...	.	56 500	57 480	51 940	88 300
34CS...	....	.31	.017	.49	.028	.13	.66	....	55 800	59 290	48 260	82 700
34CS1..	....	.31	.017	.49	.028	.13	.66	....	54 200	58 010	48 260	83 600
Memphis Bridge												
US_1MS_1 carbon.	....	23	012	.49	.027	...	...	..	41 740	43 700	40 250	64 420
US_1MS_1 Mayari.	1.58	.33	016	.64	.027	....	.37	...	62 430	72 160	52 000	99 800
U_1M_1 carbon.	....	23	.012	.49	.029	..	....		41 740	43 700	40 250	65 420
U_1M_1 Mayari.	1.58	.33	016	.64	.027	....	.37		62 830	72 160	52 000	99 800
L_1CLC_1 carbon.	....	.23	012	.49	029	....	....	....	41 740	43 700	40 250	65 420
LC_1LC_1 Mayari.	1.58	.33	.016	.64	.027	....	.37		62 430	72 160	52000	99 800

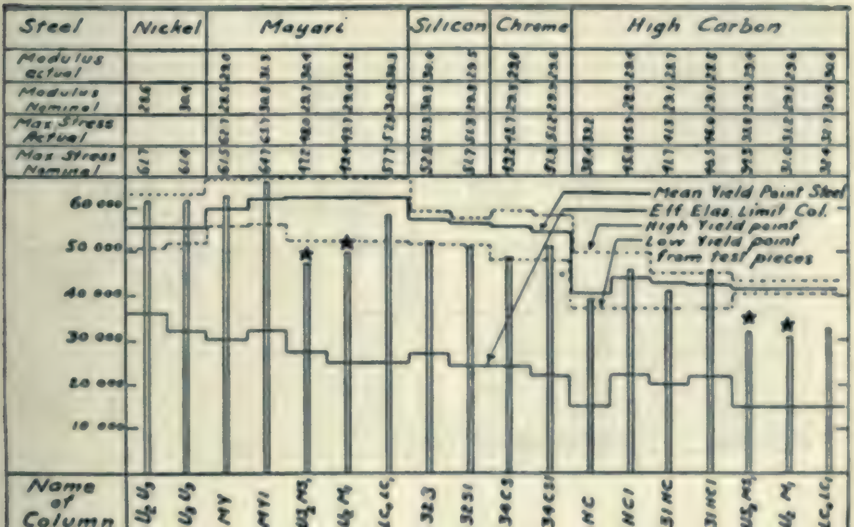
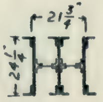
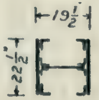
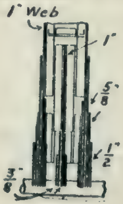
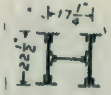
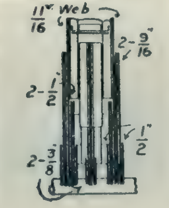
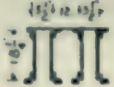
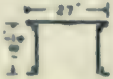
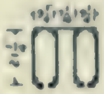
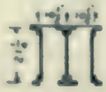
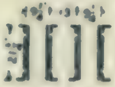
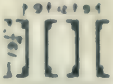


TABLE 1.—Structural and Mechanical Data Concerning Columns Tested

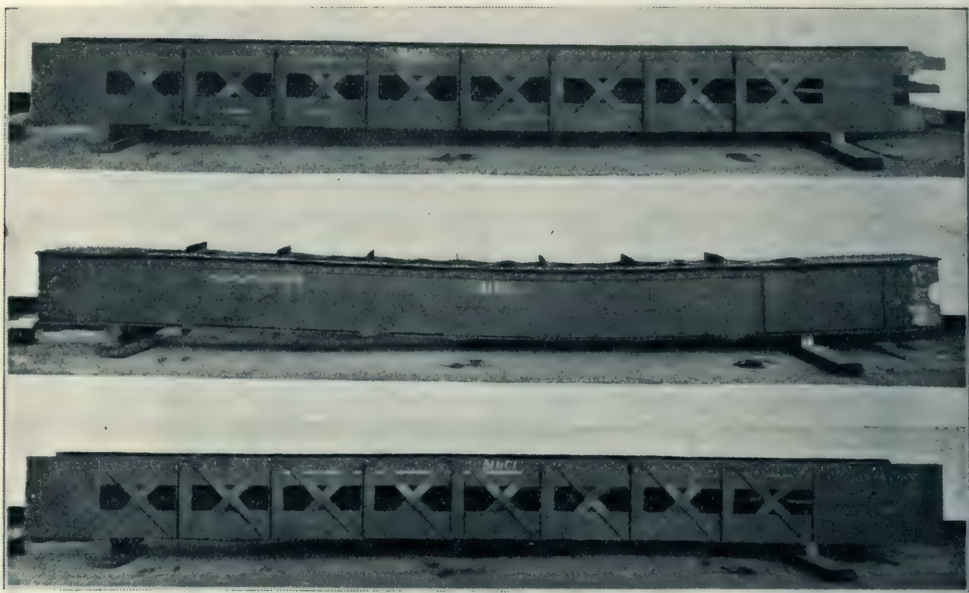
Bridge	Name of column	Steel	Manuf.	"L" length of column, center of pin to upper end.	Ends		Cross section of shaft	Material of shaft	Material at pin end	Bracing of shaft			Area, sq. in.		Radius of gyration R		Slenderness ratio= L÷R	
					Diam. of lower pin, in.	Upper				Lattice, in.	Transv. diaphgms, in.	Battens at pin, in.	Nominal	Actual	⊥ to pin	∥ to pin	⊥ to pin	∥ to pin
Municipal, St. Louis	U_2U_3	Ni	American Bridge Co.	15 ft. 8½ in. end top chord	7½	Square		2 out. webs, 24"x3/4" 4 out. Ls, 4"x4"x7/16" 1 cen. web, 24"x11/16" 4 cen. Ls, 4"x4 5/16" 2 dia. pls. 9½"x5/16" 8 dia. Ls, 2"x2"x¼"	In order from outside for U_2U_3 only 6 pls. 7/16" thick 2 pls. ½" thick 2 pls. 7/16" thick 2 webs, ¾" thick out. 2 pls. ½" thick 8 pls. 3/8" thick 1 cen. web, 11/16" thick	2½ chan. (½ web)	3 ½ thick	½ pls.	88.8	no data	7.55	8.57	25.0	22.0
	U_8U_9	Ni		24 ft. 0 in. center top chord	Square Pin not used			2 out. webs, 24"x11/16" 2 side pls. 16"x7/16" 4 out. Ls, 4"x4"x7/16" 1 cen. web, 24"x11/16 4 cen. Ls, 4"x4"x5/16" 2 dia. pls. 9½"x5/16" 8 dia. Ls, 2"x2"x¼"			3 5/16 thick	½ pls.	110.3	no data	7.78	8.97	37.0	32.1
Metropolis	HC	C	Penna. Steel Co.	All 24 ft. 4 in. upper end post	7	Square		2 side pls. 22½"x1" 4 Ls, 4"x4"x½" 1 dia. pl. 17½"x7/16" 4 dia. Ls, 3"x3"x3/8"		Transv. 4x3x5/16 in. angles diag. 4x½ bars	3 ¼ thick	¼ pls.	76.14	74.58				
	MY	My											76.14	74.90				
	31HC	C	American Bridge Co.										75.99	76.94	6.73	8.46 for	43.5 all	34.5
	32S	Si											75.99	76.94				
	34CS	Cr											75.99	76.94				
	HCl	C	Penna. Steel Co.	All 24 ft. 4 in. upper end post	7	Square		2 side pls. 22½"x11/16" 8 Ls, 4"x4"x½" 1 dia. pl. 15¾"x7/16" 4 Ls, 3"x3"x3/8"		Transv. 4x3x5/16 angles diag. 3x¼ bars	3 ¼ thick	¼ pls.	76.46	75.35				
	MYl	My											76.46	75.28				
	31HCl	C	American Bridge Co.										76.26	77.17	7.60	8.05 for	38.5 all	36.4
	32Sl	Si											76.26	77.17				
	34CSl	Cr											76.26	77.17				

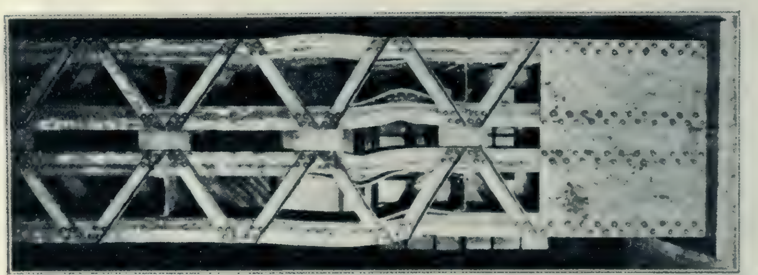
Memphis	US_1MS_1	C	Penna. Steel Co.	15 ft. 10 in. upper end post	7	Square		1 cov. pl. 27"x¾" 4 web pls. 18"x7/16" 8 Ls, 3"x2"x¾" 4 flats, 2"x¾"	Each col. rib: 1 pl. ¾" outside 1 pl. ⅝" inside 1 web, 7/16"	Transv. tie pls. ¾, see cuts; diag. bar 3x¾	4 ¾ thick	7/16 pl.	55.17	55.97	6.61	9.05	28.7	21.0
	US_1MS_1	My		15 ft. 10 in. upper end post				1 cov. pl. 27"x7/16" 2 web pls. 18"x¾" 6 Ls, 3"x3"x⅝"	Each col. rib: 2 pls. 9/16" outside 2 pls. ⅝" inside 1 web pl. ¾"	Diags. 3x1¾x¾ angles	None	⅜ pl.	42.47	41.66	6.87	9.78	27.7	19.4
	U_1M_1	C		23 ft. 6 in. upper end post	8½			1 cov. pl. 25"x¾" 4 web pls. 21"x11/16" 8 Ls, 4"x3"x¾" 4 flats, 3"x¾"	Each col. rib: 1 pl. ¾" outside 1 pl. ¾" inside 1 pl. ¾" inside 1 web, 11/16"	Diags. 3x1¾x¾ angles	6 ¾ thick	7/16 pl.	96.01	95.77	7.80	9.25	36.1	30.5
	U_1M_1	My		23 ft. 6 in. upper end post				1 cov. pl. 27"x¾" 2 out. webs, 21"x11/16" 4 out. Ls, 4"x3"x¾" 2 flats, 3"x⅝" 1 cen. web. 21"x11/16" 4 cen. Ls, 4"x3"x¾" 1 flat, 6½"x⅝"	Outside ribs (2): 2 pls. ¾" 2 pls. ¾" 2 pls. ⅝" 2 pls. ¾" } Cen. rib (1) 2 pls. 7/16" } Cen. rib (1)	Diags. 3x1¾x¾ angles	5 ¾ thick	7/16 pl.	66.55	66.02	8.17	8.47	34.5	33.8
	LC_0LC_1	C		15 ft. 6 in. bot. chord at pier				4 webs, 21"x¾" 4 side pls. 13"x¾" 8 Ls, 4"x4"x¾"	Each col. rib: 3 pls. ¾" 1 web, ¾" 1 side pl. ¾"	Tie pls. ¾ diag. 3x1¾x¾ angles	4 ¾ thick	7/16 pls.	119.00	118.18	5.74	12.14	27.6	15.3
	LC_0LC_1	My		15 ft. 6 in. bot. chord at pier				4 webs, 19"x11/16" 8 Ls, 4"x4"x¾"	Each col. rib: 3 pls. ⅝" 1 pl. ¾ 1 web pl. 11/16"	Tie pls. ¾ diag. 3x1¾x¾ angles	4 ¾ thick	¾ pls.	75.12	76.47	6.87	11.99	27.1	16.0

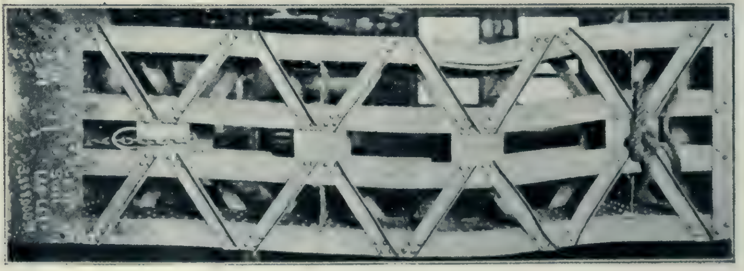
Notes on rivets: The batten plates, lattice bars, and transverse diaphragms of columns $U_3\,U_3$ and $U_9\,U_9$ were of carbon steel, the other material being of nickel steel. All of this material was subpunched and reamed. The rivets in the main members were nickel steel of ⅞ inch diameter. The rivets in the battens and lattice were carbon steel of 15/16 inch diameter. The rivets in all of the other columns were of soft steel ⅞ inch in diameter. The holes were drilled 15/16 inch diameter in the case of the Metropolis Bridge and 15/16 inch diameter in the case of the Memphis Bridge.

TYPICAL VIEWS OF FAILURES

BUREAU TESTS OF SCALE MODELS







The difficulty which the engineer encounters in making up a large section of the box or laced channel order and utilizing the definite knowledge of old test data on smaller sections is his inability to secure in the American Market a proportionate increase in the size of Angles. An 8″ × 8″ is the largest angle at present rolled by the American Mills. In the proposed Sydney Bridge an English firm contemplate rolling 12″ × 12″ × $1\frac{1}{2}$″ angles for their chord sections.

Again if the big section which is used in the full size member possesses such low elastic resistance to compression as was developed in some of the column sections tested for the Committee of the American Society Civil Engineers, since the thin metal in the scale model developed when fabricated an effective elastic limit of only about half the yield point of the individual shapes the permissible working stress of a large built up member must conservatively be smaller than that usually assumed in the design of long span structures unless the form be improved upon and a more suitable grade of metal secured.

Much of the uncertainty is involved in the adoption of sections subject to exaggerated buckling and twisting tendencies. The small section is designed with a view that it be open to inspection and painting throughout its entire area. With a large cross section there is no objection to the closed section which may be entered through a manhole, for inspection with even greater facility and convenience.

5. Economic Form of Make Up of the Compression Member can be determined only by weighing its economic advantages or disadvantages from the following standpoints:

(*a*) Cost of fabrication and percent of details of the shaft in lacing or battens.

(*b*) Economic distribution of metal permitting the highest working stress.

(*c*) Ease of making connections in the frame or truss and the satisfactory character of same.

(*d*) Dependability or safety against sudden collapse.

In the high building, from standpoints *b* and *c*, the old Phoenix column with its pintle plate connections for beam and girders may be credited first place, but the high cost of shop work with strip fillers spreading the segments for the pintle so far offsets these advantages that a Phoenix section cannot compete with the rolled *H* section which requires no shop work on the shaft of the column.

The Gray section commonly but perhaps unduly criticised as weak in bending in the light of Von Emperger's experiments on battens in lieu of lacing becomes a highly satisfactory section when filled and fireproofed with a 1-2-4 mix of concrete after erection of the skeleton frame, because its form enables the concrete to increase its strength to an extent impossible with the other sections illustrated.

In the tall building the average column length ranges from twelve to fifteen feet. Where, as in many classes of industrial buildings, the columns are two or three times this length and subjected to considerable bending, the laced section of four angles or two channels becomes in turn much more economical than the solid *H* section of smaller breadth.

Roof Construction: The mill building requires long spans but light trusses and for the compression chords of these two angles form an ideal section when supporting purlins at four to six foot intervals and panels of the triangulation of about the same length.

The details of connections with such a chord are formed with a simple gussett and the disadvantage due to the unsymmetrical form is generally less than five percent and rarely exceeds eight percent when compared with a pipe section which is an ideal form of distribution of metal for resisting compression.

The satisfactory character and simplicity of the details possible with the two angle section from the standpoint of shop work entails an economic advantage three times as great as the theoretical disadvantage in the unsymmetrical distribution of the metal in the cross section.

Compression Members for Bridges: For the vertical posts if of moderate size, two channels laced are generally the most economic form. Where the member is long and the stress relatively light, four angles laced as in (3) Fig. C7 is to be preferred. The Phoenix Section (6) of the same figure though advantageous from the standpoint that no lacing is required, and though ideal in form as far as the shaft of the column is concerned, the expense of connections has resulted in disuse.

The compression members of the Forth Bridge are illustrated in (5) of Fig. C7. Circular disposition of the metal is secured by building a tube with longitudinal plates curved and overlapping at ten points. Longitudinal ribs consisting of a plate and four angles are riveted along the longitudinal joints and stiffener frames at intervals of circular angles and plates constitute the make-up of the compression members of this structure. This section has ample radius of gyration and excellent distribution of metal from the standpoint of symmetry. The amount of work involved in the fabrication, curving and butt splicing of plates, and of squaring up the circular form at the end for main connections will undoubtedly prevent its adoption in future structures.

To obviate the disadvantages of the Forth section while securing all of its advantages the Turner section (7) of Fig. 7 was originated. It is octagon in form, made up of longitudinal plates or staves which run the full length of the section without splicing. One hundred and thirty-five degree angles inside and out join or splice the plates together. At intervals of eight to ten feet flanged cast steel rings form a frame upon which the longitudinal plates and angles are first attached and then are drilled together and riveted up. Four feet six inches is about the smallest section that it is desirable to use as it should be arranged so that the workmen can enter through a manhole for inspection, riveting and painting.

This section requires no lacing. The shop work is simple. Connections can be readily made and there are no projecting flanges or wide unsupported webs that tend to buckle or crimp out of line as in the sections customarily used in large American bridges.

Tests of models of the compression members of the Quebec Bridge disclose an ultimate strength of the compression members ranging from fifteen to nearly thirty percent below the yield point unit stress of the plates and angles which make up the section. The need of improved sections which will develop from fifteen to twenty percent higher ultimate resistance than the yield point of the individual parts is obvious.

Dependability in the compression member is to be secured by adoption of a harder metal, one that is tough but suffers only a small set as its yield point is passed, rather than a grade recommended by the rolling mill because of its lead like capacity to flow readily, receive the greatest set and fail with the most pronounced suddenness under over load in compression.

6. Temperature Stresses in the compression member made up of multiple webs like the Quebec Bridge, Blackwells Island Bridge, and others, arise from the heat of the sun's rays striking the exposed web increasing its temperature over and above that of the shaded webs. Where the metal is concentrated in large masses in the flanges it will not be heated up as rapidly as the thinner portions of the web at the central part and the metal will then be subjected to temperature stress from this cause.

Experiments made for design purposes on the Quebec Bridge are recorded in the report of the Government Board, Vol. I, page 164, as follows:

No.	Loc. of Therm.	I	II	III	IV	V
	Top	99	89	89	86	
16	Center	112	78	73	72	67
	Bottom	93	67	67	65	
	Average	101.2°	78.0°	76.3°	74.3°	
	Top	99	89	86	82	
17	Center	117	77	70	70	68
	Bottom	92	67	67	64	
		102.7°	77.7°	74.3°	72.3°	

Quoting further this report:

"Numerous assumptions might be made as to the probable temperature stresses arising under the conditions noted by the above data. If the average temperatures of the four ribs were accurately known the average temperature stresses could be calculated. But the above "averages" are not true averages. Obviously, then, the basis of calculation must be made on a probability of a certain average temperature difference between the rib exposed to the sun and the average of the other three webs. Since most of the observations were made at about 70° F. it seems only fair to assume that the probable maximum average difference at higher temperatures is at least 25° F. Accordingly this difference was assumed in the calculations. The stress variations in the different parts of the chord arising from this temperature difference were treated as secondary stresses since they do not affect the total axial stresses."

Such a section as that of the Forth bridge has a marked advantage over that of the Quebec design because of the uniform section of metal around the periphery of the chord tends by conduction to equalize the difference in temperature between the sunny and shaded side aided by the mean temperature of the air within the closed section. Whereas 25 to 30 degrees variation in temperature is to be expected for a chord of the Quebec type a third of this amount would approximate the maximum variation in the temperature of the two sides of the section of the Forth Bridge chord or one of a Turner octagon section in view of the smaller temperature variation within one web compared to the variation between different webs and the elimination of excess concentrations of metal throughout the section.

The validity of the elaborate mathematical investigation of this report hinges upon the applicability of beam theory to column bending which appears from Chapter XVIII inapplicable to column action. The readjustment of the intensity of the compression on the respective sides of the chord in the case of a symmetrical section like that of the Forth Bridge or the octagon form may be approximated by considering the maximum fiber stress increased by temperature an amount equaling two-tenths that of the expansion due to difference in temperature or treating the value of $S = 0.2$ at E[*] instead of $\frac{3}{4}$ at E[*] the approximation suggested in this report. The closed symmetrical section appears to require less than a third the allowance for temperature stress than the ordinary unsymmetrical section does.

6. Summary: The strength of a solid column may be expressed as a function of its slenderness ratio, its modulus of elasticity and a coefficient representing its end conditions. The strength of an unsymmetrical built up column can not be expressed in these terms except in an approximate manner. The secondary stresses of twisting and buckling must be considered in accordance with the law of their magnitude by separate computation.

Structural steel for bridges varying from 55,000 to 65,000 ultimate tensile strength is too soft a material to secure the most satisfactory results for compression members, particularly when a considerable portion of this tensile resistance is secured by a high manganese content. Intermediate grade steel or alloy treated steel as discussed in the previous chapter may be economically used with higher working stresses and greater safety because the compression member made up of such material will not fail with the suddenness characteristic of the rapid flow of the soft metal. In heavy sections for long span bridges the closed symmetrical section is economical because it eliminates the weight of lacing and useless details, simplifies connections, is more weatherproof and more readily inspected and painted, avoids the buckling and crippling tendency of the flanges of the box section and develops a much higher resistance and limit of elasticity for its sectional area.

[*] "a" the coefficient of expansion per degree F. = .0000065; t = temperature range 10° for the closed section, 30° for the four web sections; E the modulus of elasticity.

CHAPTER XXI

*REINFORCED CONCRETE COLUMNS

1. General Considerations. The requirements for suitable design for reinforced concrete columns in building construction may be briefly stated as follows:

First, that the longitudinal reinforcing metal should be toward the outer portion of the column in order to properly resist any tendency to bend or deflect.

Second, that the bars should be banded or tied together to maintain them in their desired position and add toughness to the column.

Third, that the bands or ties should not so cross the core as to interfere with placing the concrete and securing a monolithic solid core.

Fig. Type A, shows one of the old forms of Hennebique type of column. In this type the principal reinforcement consists of heavy vertical or longitudinal bars tied across laterally from one to the other by comparatively small ties. It will be noted that these ties cross and recross the core of the column, and require considerable care in filling to make sure that there are no voids in the finished work.

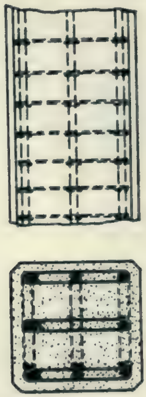


Fig. Type A Column

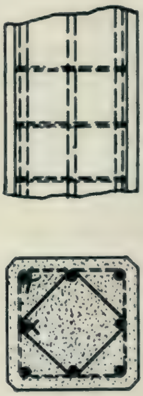


Fig. Type B Column

*First published in Concrete Steel Construction by H. T. Eddy and C. A. P. Turner.

Cases have occurred with this type where the concrete has been arrested part way down in pouring the column and on removal of the form an open space was found of perhaps as much as two feet between the concrete above and below, so that the load above was carried by the vertical bars only. Evidently such an arrangement of metal is somewhat dangerous, but with unusual care it may prove satisfactory from the standpoint of strength if the work be properly executed.

Fig. Type B, shows an improved form of tying together the eight vertical bars forming the vertical reinforcement with horizontal ties in the form of squares, one inscribed within the other. The advantage of this type over that previously shown lies in the fact that the central core of the column, or inscribed square, is clear and unobstructed thruout.

Fig. Type C, shows a column reinforcement consisting of four vertical rods with wrapping or ties holding them together at intervals. This is suitable for very light loads where the concrete is more than sufficient to take the entire compression without excessive stress.

Fig. Type D, shows a column section of the Considère type in which the vertical rods are hooped with spiral reinforcement. Considerable work has been executed using hooped columns that omit the vertical steel. This, as the authors view it, is a very grave mistake.

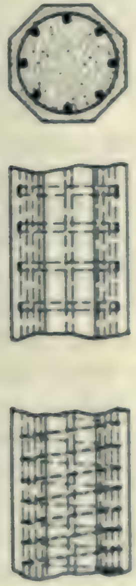


Fig Type D Column Considere Type

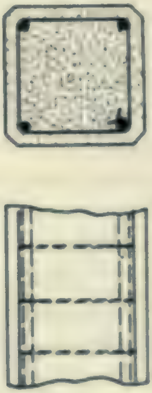


Fig Type C Column Suitable for Light Loads Only.

Hooping may be of two types: First, a spiral coil in which the wire is wound around the core of the column in the form of a continuous spiral, and second that in which separate independent hoops are placed at intervals and attached to the vertical reinforcement.

The strength of Types A, B, C, and D, all depend, first upon the strength of the concrete, second, upon the amount of vertical steel used, and, third, upon the amount of ties or hooping holding the rods in position and bringing lateral restraint upon the concrete.

Theoretical formula based on the ratio of the moduli of elasticity of concrete and steel alone cannot be depended upon for a satisfactory solution of the problem presented by the third element noted and we must depend largely upon experimental investigation to determine reasonable and safe practical values to use for our working stress.

In deciding upon these values we need to consider the column, first, from the standpoint of its ultimate strength in the finished building, second, from the standpoint of its strength and safety during construction, and third, from a consideration of the relative values of the various types in securing strength at a minimum cost.

Type D, with a proper proportion of vertical steel combined with the hooping ranks first, from the standpoint of safety and economy.

Type B, second.

Type D, with hooping but with no vertical steel third; and types A and C fourth.

It may be stated that type A is now rarely used and discussion concerning it may be omitted.

For Type C, the allowance permissible for working stress on a 1 : 2 : 4 concrete is 350 pounds per inch of the core area between rods, and 10,000 pounds per square inch on the vertical steel and besides this the volume of metal in the ties is to be treated as forming imaginary verticals with a working stress equal to that allowed for the vertical bars, the ties to be spaced not further apart than ten times the diameter of the vertical bars in case the bars are one inch section, but where smaller bars are used the spacing should not exceed 9″ nor the size of the tie to be less than one quarter inch round.

Type B. The allowable working stress for a 1 : 2 : 4 mix is 600 pounds per square inch on the concrete of the core, 10,000 pounds on the vertical steel, and one and one half times the volume of the ties treated as imaginary verticals. These ties should not be spread further apart than 9″, and, if they are to be considered of value, they should be put not more than ten diameters of the vertical bars apart.

Type D, the Considère type, is by far the most economical type of column reinforcement that has been invented. It was brought prominently to the attention of the public by Armand Considère.

The principle involved is this: by restraining the concrete laterally its strength in compression is greatly increased. Just as an ordinary piece of stove pipe filled with sand will carry a load several times greater than the pipe itself would be able to do, so will a hooped column owing to the fact that the metal is strained in tension, while the filling held in position by the restraint of the pipe, carries the weight of the load. For strength see Section 5.

There have been quite a number of experiments on hooped concrete using spiral hooping only In these experiments it has been found that after the ultimate strength of plain concrete has been developed, splitting and scaling of the outside shell occurs, combined with a large vertical deformation and considerable lateral bending before ultimate failure.

2. Considerations of Safety Determining Carrying Loads. If it is expected to develop the core of the concrete to a point beyond its normal strength we must evidently prevent its lateral distortion or bulging and also the sliding or flow of the concrete between consecutive bands or turns of the spiral, hence a certain proportion of vertical steel must be used in connection with the hooping to secure the best results.

In determining the degree of safety of the various types of column design, an investigation of the manner of failure of the respective types is in order as to whether it occurs suddenly and without warning, or gradually, accompanied by indications of approaching failure long before failure occurs, and there is the further question as to whether the conditions of strain in the column are proportional or comparable between the column under ordinary working stresses and the column as it approaches the breaking down point and ultimate strength. The following general observations may be made as to these questions:

In columns shown in Fig. Type C, the failure occurs with little warning, the vertical bars bending outward and the ties yielding.

In the hooped column without vertical steel, when it is loaded from forty to fifty percent of its ultimate strength, the portion of the concrete outside the hooping commences to check and crack, and later to scale. From this point, the rate of deformation with addition of the load increases rapidly owing to dissipation of energy by the cracking and scaling of the shell. Further loading is accompanied by large lateral deformations up to the final failure. Such a column gives ample warning but the point at which the outer shell or fire protection commences to fail is but little higher than the point at which the ordinary vertically reinforced column fails, so that little advantage in the way of increased working stress is secured unless the hooping is combined with vertical steel.

The well hooped column vertically reinforced shows a large increase in strength over that of the vertically reinforced column with ties and a great increase in toughness. Its failure is not sudden and without warning as in the former type, while the point at which checking and scaling of the outside shell occurs is raised to eighty or eighty-five percent of the ultimate strength, thus giving a large margin of safety to the fireproofing between the working load and the load where the failure of the shell is in evidence.

3. Experimental Data. A partial report on tests on full sized columns, made at Phoenixville, Pa., for C. A. P. Turner, engineer, by Mason D. Pratt, is given in the following table:

Test No. 1

Marks on column: None.

Reinforcement: Eight $1\frac{1}{8}$ inch round vertical bars.

Bands: Spaced 9 inches vertically, $\frac{1}{2}$ inch rivets, cross section $1\frac{3}{4}$ x $\frac{1}{4}$ inches, inside diameter 14 inches.

Hooped with 7/32 inch wire spirals, about 2 inch pitch.

Total load at failure, 1,360,000 lbs.

Remarks: Point of failure was about 22 inches from the top, Little indication of failure until ultimate load was reached.

Some slight breaking off of concrete near the top cap, due possibly to the cap not being well seated in the column itself.

Test No. 2

Marks on columns: Box 4.

Reinforcement: Eight $1\frac{1}{8}$ inch round vertical bars.

Bands: Spaced 13 inches vertically, $\frac{1}{2}$ inch rivets, cross section $1\frac{3}{4}$ x $\frac{1}{4}$ inches, inside diameter 14 inches.

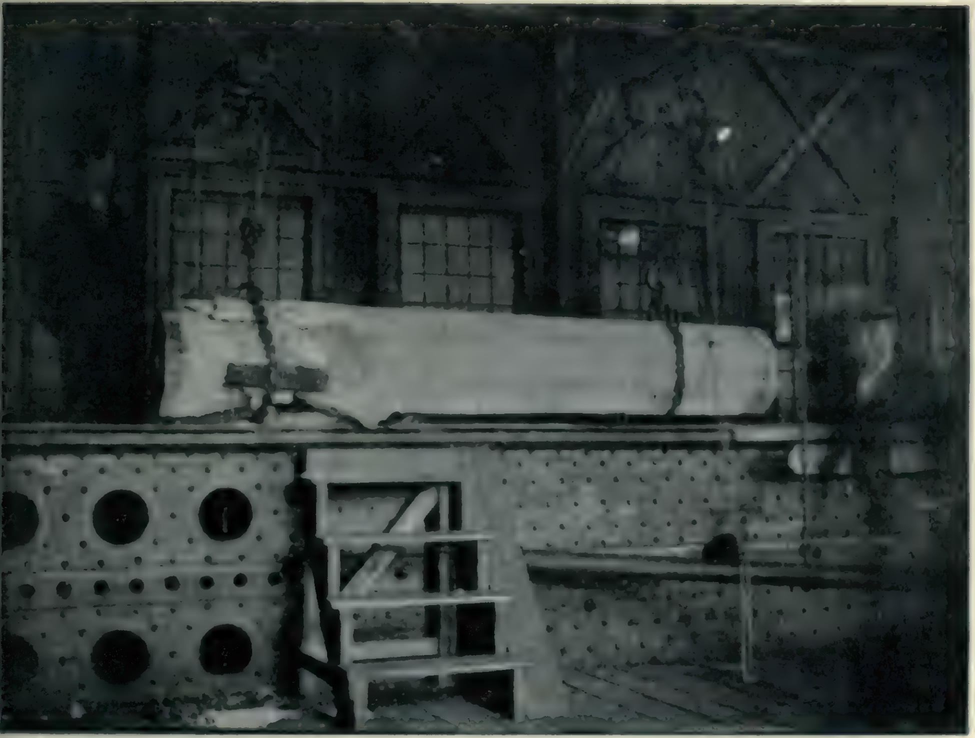
Hooped with 7/32 inch wire spiral, about 3 inch pitch.

Point of failure: About 18 inches from top.

Top of cast iron cap cracked at four corners.

Ultimate load: 1,260,000 lbs.

Remarks: Both caps apparently well seated, as was the case with all the subsequent tests.



Test No. 1
Showing Column as it Came from Testing Machine.

Test No. 3

Marks on column: 4-b.

Reinforcement: Eight 7/8 inch round vertical bars.

Bands: Spaced 13 inches vertically, ½ inch rivets, cross section 1¾ x 3/16 inches, outside diameter 14 inches.

Ultimate load: 900,000 lbs.

Point of failure: About 2 feet from top.

Remarks: Concrete at failure, considerably disintegrated, probably due to continuance of movement of machine after failure.

Test No. 4

Marks on columns: Box 4-c.

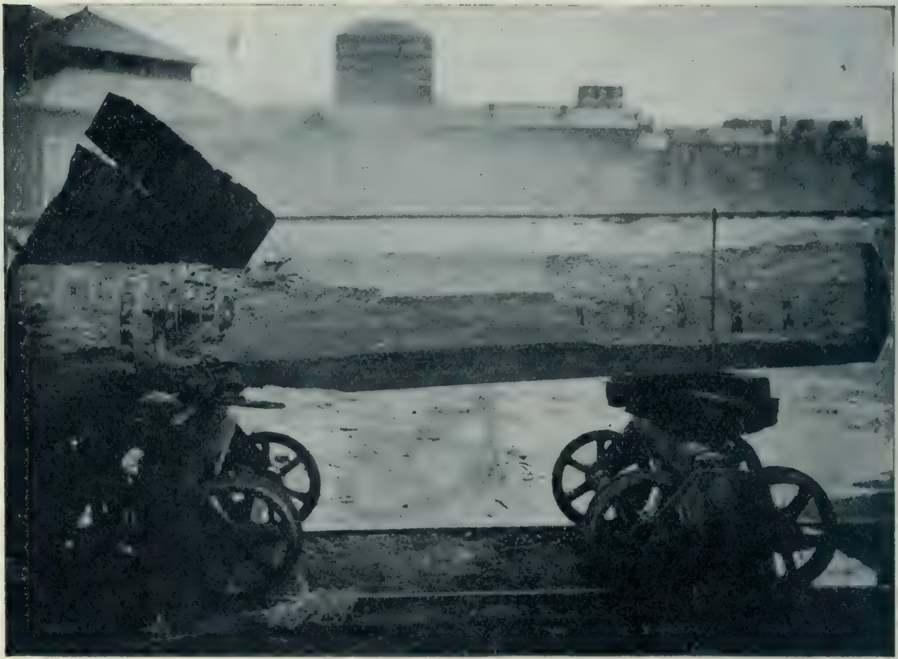
Reinforcement: Eight 1 inch round vertical bars.

Bands: Spaced 8 inches vertically, $\frac{1}{2}$ inch rivets, cross section $1\frac{3}{4} \times \frac{1}{4}$ inches, inside diameter 14 inches.

Hooped with 7/32 inch wire spirals, about 3 inch pitch.

Total load at failure: 1,260,000 lbs.

Remarks: First indications of failure were nearest the bottom end of the column, but the total failure was as in all columns, within 2 feet of the top. Large cracks in the shell of the column extended from both ends to very near the middle. This was the most satisfactory showing of all the columns, as the failure extended over nearly the full length of the column.



Column No. 4, after Test

Test No. 5

Marks on column: None.

Reinforcement: Eight $\frac{7}{8}$ inch vertical bars.

Bands: Spaced 10 inches vertically, $\frac{1}{2}$ inch rivets, cross section $1\frac{3}{4} \times \frac{1}{4}$ inches, $14\frac{1}{2}$ inches outside diameter.

Hooped with 7/32 inch wire spiral as before, 3 inch pitch.

Load at failure: 1,100,000 lbs.

Ultimate load: 1,130,000 lbs.

Remarks: The main point of failure in this as in all other columns was within two feet of the top altho this column showed some scaling off at the lower end.

This set of tests were not conducted with any considerable degree of refinement but were a practical test of ultimate strength and the yield point value of specimens of full sized members, which lends greater value to the determination than laboratory tests on small specimens.

The concrete mixture was one part Portland cement, one part sand, and one and one-half parts buckwheat gravel, and three and one-half parts gravel ranging from one-quarter inch to three-quarter inch in size.

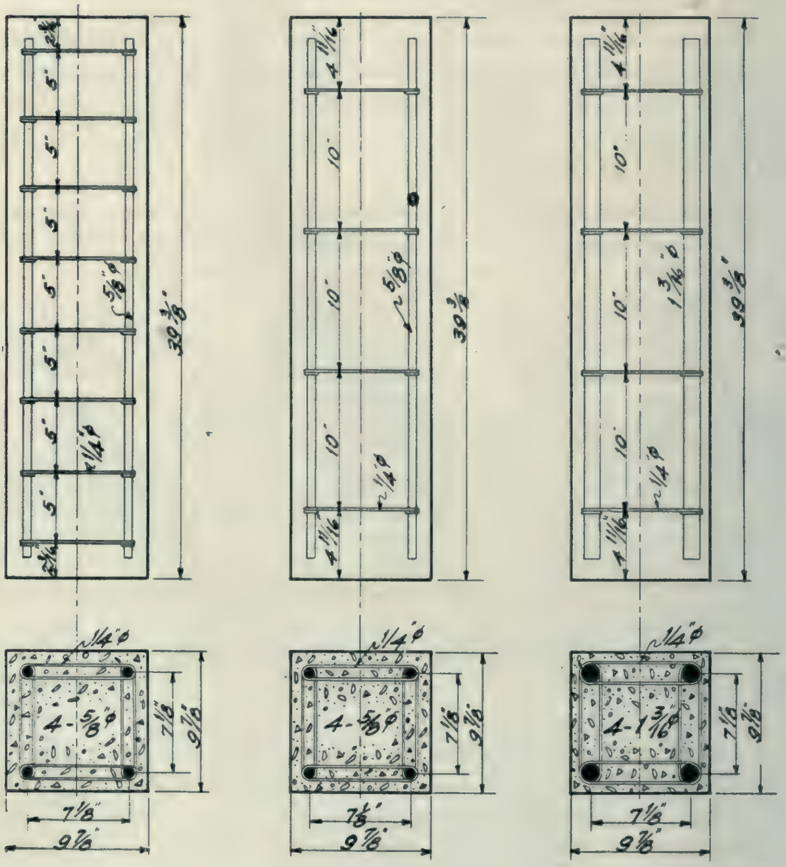
It should be noted that in these tests the cracking of the shell did not occur until the hoops were over-strained, and that the strength of the hooping closely defined the ultimate strength of the column with the proportions of vertical steel used.

1. In these columns, pressures were developed on the core more than three times the ultimate strength of plain concrete at 2600 pounds per square inch.

2. Incipient failure occurred only by the stretching or bursting of the bands.

All columns were approximately octagonal in shape, 10′6″ long and 18″ diameter. Final failure occurred toward the upper end of the column. Mr. Pratt accounts for the regularity with which the columns failed at the upper end on the ground that the concrete at the lower end was more dense, owing to its being under considerable hydraulic pressure, while setting. The rods were all shorter than the concrete shaft. Examination of the columns after removal from the testing machine showed in all cases a bulging out of the vertical reinforcement at the principal point of failure with the nearest hoop ruptured and in every case the wire spiral was broken in one or more coils at the point where the vertical rods were bent out. The vertical bars in nearly every case bulged as a column with fixed ends. Where the hoop spacing was six to nine inches, the deformed length of the bar would extend over the space of two hoops. Where the hoop spacing exceeded nine inches the deformed length of vertical bars was confined to the space between one pair of hoops.

It may be well to review at this point two series of carefully conducted tests upon plain and reinforced concrete prisms. Fig. 127, shows the dimensions and reinforcement of a series of prisms tested by Bach* for Wayss and Freitag. These specimens are Type C columns.



Test Specimens

Fig. 127 Showing Dimensions and Reinforcement.

The first table for elasticity test of Type C columns, shows the reinforcement, tie spacing, and elastic deportment under different loads.

The second table for Type C columns, shows the breaking strength of this series.

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*See Het Cement-Ijzer, Saunders. p. 91.
Eisenbetonbau, Mörsch.

It will be noted under the table showing the breaking strength that the specimens with 1 3/16 inch round verticals and 4.6 percent reinforcement, do not show as great an increase in strength as the specimens with 1.14 percent reinforcement, but with much closer spacing of the ties, the specimens with the large rods having ties four times as far apart as the specimens with the small rods.

COL. TABLE I

Elasticity Test of Columns (Bach)

Stresses	4 Rods of Diameter	Tie Spacing	Shortening in Millionths of the length		
Lbs. sq. in.	Inch	Inches	Total	Elastic Diff.	Permanent Set
459	Plain Concrete		133	7	126
459	⅝ in. rd.	10	114	5	109
459	⅝ " "	5	110	2	108
459	⅝ " "	2½	106	4	102
919	Plain Concrete		333	37	296
919	⅝ in. rd.	10	267	20	247
919	⅝ " "	5	264	18	246
919	⅝ " "	2½	241	13	228
1380	Plain Concrete		709	164	545
1380	⅝ in. rd.	10	488	63	425
1380	⅝ " "	5	473	58	415
1380	⅝ " "	2½	421	42	379

COL. TABLE II.

Breaking Strength of Columns (Bach)

Specimen 3 mo. old.		Breaking Strength				
Diameter of rods.	Tie Spacing	Each			Average lbs. per sq. in.	Percent of Reinforcing
Plain	Concrete	2076	1963	1977	2006	0
⅝ in. rd.	10 in.	2432	2290	2447	2390	1.14
⅝ " "	5 "	2389	2660	2489	2512	1.14
⅝ " "	2½ "	3015	2845	2887	2915	1.14
[illegible] " "	10 "	2404	2404	2446	2418	2.04
1 3/16 " "	10 "	2474	2830	2802	2702	4.60
Test Cubes.		2389	2404 2631	2432 2617	2490	0

Looking at the elasticity tests of the specimen reinforced with the same vertical steel the marked difference in toughness, increase of modulus of elasticity and the reduction of permanent set produced by the increase in the number of ties, is strikingly shown by this series of tests.

This series of tests show conclusively that where the vertical bars do not bear directly upon the face plate of the machine and the load is brought upon the reinforcement under the usual condition of the column in the reinforced concrete structure, that is the steel is strained by the load brought on it thru the concrete, that a formula which does not take into consideration the amount and spacing of the ties fails to account for the deportment of the column both as regards elasticity and ultimate resistance. That is, merely an increase in the total cross section of the longitudinal reinforcement does not produce an increase in the breaking strength to the extent which would be expected by the formula

$$P = f_c (A_c + nA_s)$$

Hence in inexperienced hands, this formula may produce unsafe designs by increasing the percentage of longitudinal reinforcement disproportionately in order to secure columns of small diameter. This procedure gives a column with a calculated margin of safety which it does not possess.

When the increase in the resistance is compared for equal longitudinal reinforcement, there being a difference in the number of ties, it is shown by these experiments that the steel used as ties is much more effective than the longitudinal steel. As noted, however, the columns reinforced with vertical steel and ties do not develop that degree of strength which is secured by columns properly reinforced vertically and hooped with bands or spirals. Hence their use has been largely discontinued where loads are at all heavy and is confined to those cases where loads are relatively light and some slight saving may be made in the use of ties over the cost of spiral hooping.

4. Reinforced Columns Classified by the Manner in Which Loads are Applied. We have called attention to the type of prisms which were tested by Bach in which the steel was cut short at the end of the concrete prism so that the load applied to the prism was brought upon the longitudinal steel thru the bond shear between the steel and the concrete. This is the common mode of arrangement of the steel in the reinforced concrete column in the practical building. Sometimes, however, in order to secure relatively small columns for heavy loads very high percentages of vertical steel are used and the load is brought largely upon the steel by direct bearing of steel upon steel. Evidently the coaction of the elements of the composite structure differ widely under these radically different conditions and correspondingly different formulas must be applied to these different conditions.

We have cited the interesting series of tests by Bach on rectangular prisms, plain and reinforced with longitudinal steel and ties and we may profitably consider the corresponding series of tests on hooped and longitudinally reinforced prisms.

Fig. 128 gives the dimensions and type of reinforcement used while table III shows the results of the tests.

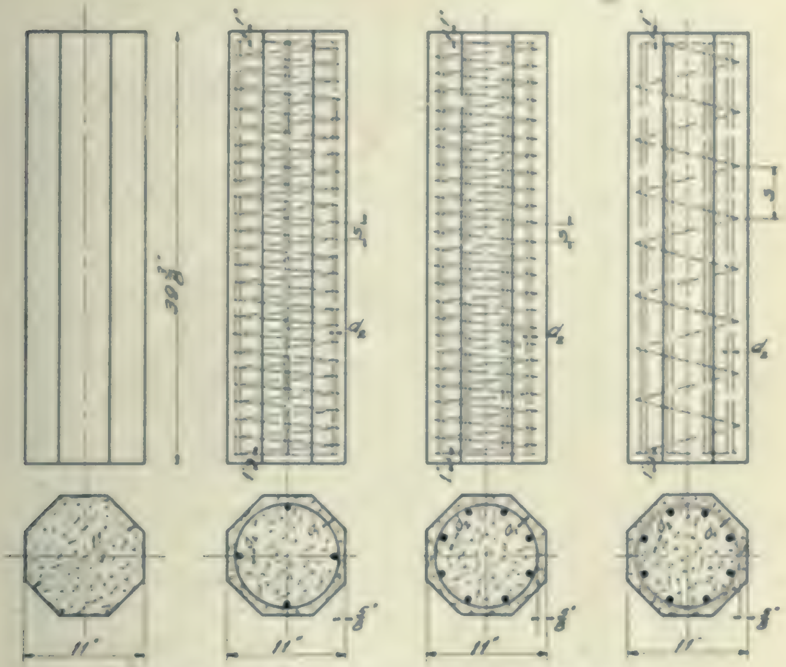

Fig. 128 Dimensions of Columns and Type of Reinforcement.

As in the first series the longitudinal steel was cut short of the ends of the prisms so as to secure normal coaction between the steel and the concrete. Percentage of hooping varied thru a wide range and the pitch of the spiral hooping was also varied between wide limits. Unfortunately the series is lacking in a corresponding variation in the percentage of vertical steel.

On the whole, these specimens correspond approximately with results indicated by the Considère formula and exceed these results for those specimens which approach desirable proportions in point of spacing of the spiral and relation of the vertical steel to the hooping.

COL. TABLE III

Bach's Tests of Plain and Hooped Concrete Prisms, Longitudinal Steel Cut Short of End of Prism.

Col. No.	Description	Spirals		Verticals		Area Core sq. in.	% Reinforcement			Load lbs. per sq. in. when first cracks started.	Increased st. due to reinf. Lbs. per sq. in. at yield pt.	Increased st. lbs./ sq. in. per 1% reinf. at yield pt.	Ultimate strength lbs. per sq. inch	Increased ult. st. lbs. / sq. in. due to reinf.	Increased ult. st. in. lbs. per sq. in. per 1% reinf.	Load when considering core only lbs./ sq. in.
		Dia. wire in.	Pitch	No.	Dia. in.		Spiral	Vert.	Total							
I	4 spec.	Unr	einfor	ced			0	0	0	1,890			1,890			
II	3 "	.2	1.52	4	.28	69.69	.88	.35	1.23	2,260	370	301	2,260	370	301	3,270
III	3 "	.28	1.48	4	.28	68.51	1.78	.36	2.14	2,290	400	187	2,550	640	299	3,660
IV	3 "	.4	1.68	4	.28	66.76	3.24	.37	3.61	2,420	530	147	3,420	1,525	422	4,930
V	3 "	.2	1.52	8	.44	69.69	.88	1.75	2.63	3,190	1,300	495	3,210	1,320	502	4,650
VI	3 "	.28	1.48	8	.44	68.51	1.78	1.77	3.55	3,270	1,380	389	3,270	1,380	389	4,720
VII	3 "	.4	1.72	8	.44	66.76	3.17	1.82	4.99	3,460	1,570	315	4,000	2,110	424	5,770
VIII	3 "	.28	1.24	4	.28	68.51	2.12	.36	2.48	2,790	900	363	2,850	960	387	4,100
IX	3 "	.40	1.60	4	.28	66.76	3.41	.37	3.78	2,420	530	140	3,000	1,110	294	4,340
X	3 "	.48	1.64	4	.28	65.61	4.82	.38	5.20	2,560	670	129	3,640	1,750	336	5,260
XI	3 "	.56	1.48	4	.28	64.47	7.36	.38	7.74	2,250	360	47	3,500	1,610	208	5,050
XII-I	3 "	.28	1.60	8	.20	68.51	1.64	.37	2.01	2,320	430	214	2,320	430	214	3,360
XII-II	3 "	.40	1.60	8	.28	66.76	3.41	.74	4.15	2,330	440	106	3,270	1,380	333	4,720
XII-III	3 "	.56	1.60	8	.40	64.47	6.82	1.56	8.38	2,620	730	87	4,290	2,400	286	6,200
XIII-I	3 "	.28	3.20	8	.28	68.51	.83	.72	1.55	2,300	410	264	2,300	410	265	3,330
XIII-II	3 "	.40	3.20	8	.40	66.76	1.71	1.51	3.22	2,550	660	205	2,580	690	214	3,720
XIII-III	3 "	.56	3.20	8	.48	64.47	3.43	2.25	5.68	2,650	760	134	2,830	940	166	4,240
XIV-I	3 "	.28	4.80	8	.40	68.51	.55	1.47	2.02	2,200	310	153	2,210	320	158	3,190
XIV-II	3 "	.40	4.80	8	.48	66.76	1.15	2.17	3.32	2,600	710	214	2,600	710	214	3,750
XIV-III	3 "	.56	4.80	8	.56	64.47	2.30	3.11	5.41	2,950	1,040	193	2,940	1,050	195	4,250

Specimens had an octagonal section with a diameter of 10.8 inches and had a length of 39.37 inches. Mix—1 part of Heidelberg Portland Cement to 4 parts of Rhine Sand and gravel. Specimens 5 to 6 mo. old when broken.

Specimens XIII and XIV, in which the pitch of the spiral was exaggerated gave results less than the formula, pointing to the necessity for closer spacing.

In specimens II, III, IV, VIII, IX, X, XI and XII, the sectional area of the longitudinal rods was small and the results were consequently indifferent, but the greater the total weight of spiral reinforcement the higher were the values.

The tests show that when the spirals are increased in strength, their pitch must be decreased and the cross section and number of vertical rods must be increased, for with the increase of spirals the concrete is in a condition to resist heavier pressure and its tendency to force its way out between the longitudinal rods and the spiral bands increase.

Examining the table of the values obtained, the increase in strength and yield point value per one percent of total steel is found to be greatest in column specimen V, in which the spiral reinforcement is .88 percent and the vertical reinforcement was 1.75. In columns XII and XIII, with the spiral reinforcement 6.82 percent and the vertical reinforcement 1.56 percent, the increase percent of total reinforcement at the yield point was only 87 pounds and the increase of the ultimate strength per percent of the ultimate reinforcement 286 pounds per square inch, showing an improper proporcion of the spiral and vertical reinforcement.

On noting the pitch of the spiral it is quite noticeable that the specimen with a small pitch gives the highest resistance per percent of steel. In none of these specimens, however, are the longitudinal rods of sufficient size to secure the best results. Had 5/8, 3/4 and 1 inch rods been used in place of 1/4 inch, 7/16 and 1/2 inch, much higher values would have resulted from the stiffness of the verticals which must act as beams from coil to coil in resisting outward or bulging pressures.

The following table gives the percentage of vertical steel and hooping in the Phoenixville tests, see Section 3.

COL. TABLE IV

Phoenixville Column Tests. Specimens 3 mo. old

No. of Test	% Verticals	% Spiral	% Hooping	% Spiral and Hooping	% Total	Ultimate Tested Strength	Ultimate Strength Per Sq. In.	Computed Increase per % of Steel	Strength by Considère Formula
1	5.17	1.54	.99	2.53	7.70	1,360,000	8,850	785	1,291,000
2	4.82	1.35	.66	2.01	6.83	1,260,000	7,630	710	1,279,000
3	3.12	0	.72	.72	3.84	900,000	5,800	792	898,300
4	4.08	1.36	1.12	2.48	6.56	1,260,000	8,200	820	1,216,000
5	3.12	1.36	.90	2.26	5.38	1,130,000	7,350	842	1,127,000

From these tests it may be observed that the best results are to be secured when the value of the vertical steel as an element of strength is approximately the same as the hooping according to Considère's formula, Section 5. In other words, the reinforcement should be so proportioned that the volume of the hooping for this type of column should be between the limits of 35 and 45 percent of the volume of the vertical steel to secure the best results in increasing the yield point and ultimate strength of the column. Between these limits the yield point value of the columns should vary from 75 to 85 percent of the ultimate strength of the specimen, so that due warning of approaching failure is given by the member while ample margin exists between the safe working stress and that point at which the fireproofing will commence to scale and chip.

5. Considère Formula. As we have pointed out, the Considère formula is conservatively applicable to the vertically reinforced and hooped columns provided that the hooping and vertical steel are properly proportioned. With no vertical steel, the yield point value of the column is not sufficiently raised to warrant very material increase in safe working stress over that of ordinary types notwithstanding the fact that there is an increase in the ultimate resistance and large deformations occur before ultimate failure.

The Considère formula for column resistance is as follows:

$$P = 1.5\,A_c f_c + (f_s A_s + 2.4 f_s' A_s')$$

in which A_c is the area of the core, A_s the area of the longitudinal steel, A_s' is the area obtained by dividing the volume of the hooping by the length of the column. The coefficient 1.5 is a coefficient which Considère considers represents the effect of the hooping in increasing the strength of the core, and that this coefficient is a maximum at 1.5 and that it is less than this value for a percentage of hooping which does not furnish a resistance equivalent to 700 pounds per square inch lateral pressure on the column.

Mörsch, however, and also Saunders, in discussing the Bach tests where the percentage of steel was low, seem to treat this coefficient of Considère as the ratio of the core to the total area of the column (fireproofing and all). In most building ordinances this coefficient, however, is taken as unity. 2.4 times the volume of the hooping is figured as the effect of the stress on the hooping in increasing the crushing resistance of the core by the lateral pressure brought about by the stress in the hooping and represents the resistance which would be developed were the hooping in the form of cylinders and filled with a granular mass such as sand subjected to pressure.

Hence $2.4A'_s$ equals the cross section of the equivalent imaginary verticals.

Le Genie Civil, Feb. 9, and 16, 1907, reported a very extensive series of tests by Considère on 260 columns with lengths varying from 3.15 in. to 13 ft. 1½ in. and diameters from 1.8 in. to 27.5 in. The percentage of reinforcement varied from 1 to 14, various methods of spiral reinforcement, including concentric spirals were tried. The effects of richness of mixture, age, percentage of water, ramming and irregularities in workmanship were also studied. One specimen having 12.9 percent spiral and 1.2 percent longitudinal reinforcement, and made from a mixture containing 1830 lb. of cement per cu. yd. of sand sustained 25,600 lb./in.^{2} at rupture. A similar specimen having only 6.6 percent of spiral reinforcement failed at 17,200 lb./in.^{2} at rupture, with a deformation of 12 percent. From these and previous tests Considère deduced the conclusions that the rupture load of a spirally reinfored concrete column exceeds the sum of the three following factors:

1. The strength of the plain concrete times 1.5.
2. The strength of the longitudinal rods stressed to their yield point.
3. The strength of a longitudinal reinforcement of 2.4 times the weight of the spiral stressed to its yield point.

Turner, in the design and guarantee of strength for working severa hundred thousand columns has used the Considère formula slightly modified,allowing 800 pounds per square inch on a 1: 2: 4 mix, 12,000 pounds per square inch on the vertical steel, 16,000 pounds per square inch on the hooping, treating the spirals as imaginary verticals having a volume 2.4 times the volume of the hooping. For these values the ratio of the length of the column to its diameter should not exceed 12. For a longer column increased vertical steel should be used to resist flexure. Where the vertical resistance of the concrete developed by the hooping exceeds 2000 pounds per square inch the proportion of the mix should be increased from 1 : 2 : 4 to 1 : 1½ : 3, and extra care should be used in the selection of the stone aggregate to see that it is hard and satisfactory. Screened gravel is preferable where high working pressures are used in the columns. To be efficient, spiral hooping should not exceed a 4″ pitch when combined with not less than four vertical rods and these should be limited to a minimum diameter of 3/4 of an inch for columns carrying moderate loads, or to a pitch not exceeding 3″ if the full value above recommended is to be used in considering the hooping. A reduction of the allowance on the vertical steel should be made where the spacing of the vertical bars exceeds 9″ center to center in proportion to fifty percent of the increase in spacing of the bars.

6. Safe Ultimate Limit of Compression. A wonderful degree of strength which may be developed by properly hooped and longitudinally reinforced concrete columns is established by the Considère experiments and it becomes a question as to how great values it is permissible to use. Turner has found it permissible, where small diameters are desired, to use as the approximate limit of safe design a working stress as high as 4000 pounds per square inch of cross section of the core including the vertical steel. Under these ultimate conditions it is recommended that the column should have enough vertical steel to carry the entire load at little more than the yield point value, say at 50,000 pounds per square inch: that there be sufficient hooping to develop the value of the load figured at

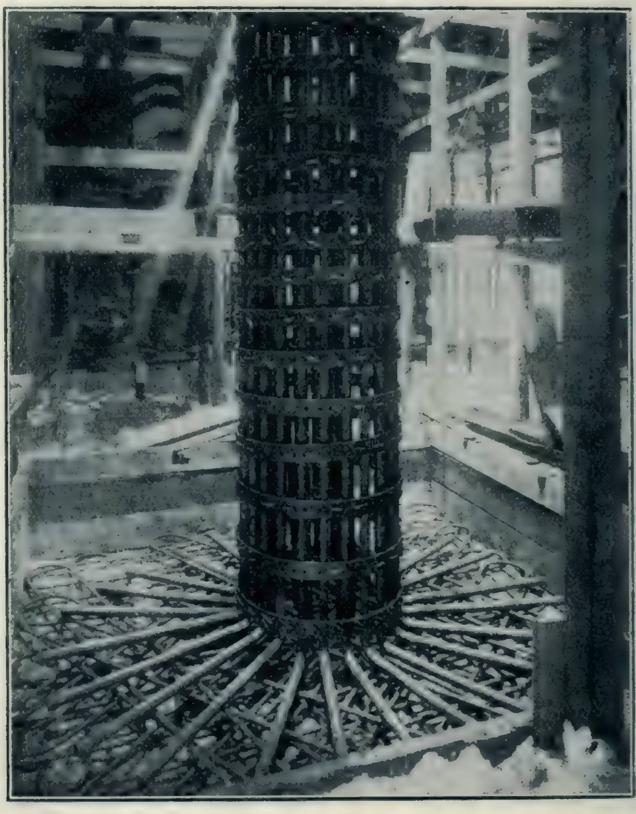

Fig. 129. Cut showing reinforcement of column carrying working pressure of 4000 pounds per inch on the core.

40,000 pounds per square inch on imaginary verticals corresponding to 2.4 times the volume of the hooping; and that the gross area of the column inside the fireproofing should be sufficient to carry the load at 4000 pounds per square inch.

7. The Mode of Operation of the Reinforcement in Concrete Columns. In a practical concrete structure, as a rule, the load is brought on the column thru the beams or the concrete of the slab and the load is transferred to the vertical steel thru the medium of the concrete by the bond between the concrete and the steel.

In the tests by Withey and those of some other American investigators, the load on the test specimens was transferred directly to the steel by direct bearing between the longitudinal reinforcement and the face plate of the testing machine. This makes an important difference in the mode of operation of the column. The vertical steel is a more rigid element under compression than the concrete and if the load be not applied directly to the steel but be brought on the steel thru the surrounding matrix, the coaction of the two materials is brought about by bond shear and indirect stress generated by the bond shear between the steel and the concrete. Now, since there is no slipping of the steel in the concrete within the yield point of the column, the same amount of potential energy or work of deformation is stored up by the indirect stress of the bond shear between the concrete and the steel as is stored directly in the steel. Hence it appears that in this case the steel by help of bond shear operates to store energy more efficiently up to the point where cracks and checks occur in the concrete than a mere comparison of the relative moduli of steel and concrete would indicate.

This phase of the co-operation of the steel and the concrete in the column requires, for its greatest efficiency, a strong, rich, concrete, in order that the most dependable bond may be secured.

If, however, the load be brought upon the steel by a rigid bearing between the face plate of the testing machine and the steel of the longitudinal reinforcement and in like manner on the concrete core, such a bearing makes the elastic deformation of the steel the same as that of the concrete thruout, and no elastic efficiency by coaction of the concrete and metal exists, such as occurs in the practical structure.

8. Effect of Hooping. The crushing of solid bodies cannot take place without lateral swelling. Therefore, by resisting any such swelling, the compressive resistance of the column is increased. In the practical column this resistance to lateral bulging or swelling is furnished by the hooping in which the spacing should be limited generally to four inches for light pressures and closer spacing for higher pressures. As the hooping is brought into play by lateral swelling or bulging, the degree of restraint furnished by the hooping alone is not uniform and this lack of uniformity causes the concrete to check and crack outside of the hooping under pressures as large approximately as the ultimate strength of plain concrete.

The addition of vertical steel distributes the bulging pressure from band to band or hoop to hoop to such an extent that the vertical steel which has been added forms beams spanning the spaces between coils or bands and does so to an extent measured by the more or less close spacing of the vertical bars. In other words, the vertical reinforcement receives lateral pressures between the hoops and transfers it to the hoops as supports. This action destroys the equilibrium of uniform pressure outward upon the hoops and tends to deform the configuration of the circular hoop from circles to polygons having apices at the point of bearing of the vertical steel against the hoops. This action between the vertical steel and the hooping induces indirect stresses between the hoops and the vertical bars similar to those in the slab with two way reinforcement.

The outer shell of the concrete is subjected to direct compression vertically and circular tensions horizontally, brought about by the bulging tendency. These circular tensions are reduced between verticals by the compression brought about by the tendency to change configuration in the hooping just mentioned, while the ring tension opposite verticals is increased by this action. The lateral reinforcement by its compressive action exerts a powerful effect to resist bulging and to prevent circumferential elongations so that the reinforcement (both lateral and longitudinal), enables it to withstand much greater deformations without cracking or checking than would be otherwise possible.

Let us review the action of the column as the load is applied with reference to the manner in which the potential energy of internal work is stored within the structure. As the load is gradually applied we have a certain elastic deformation and the internal work is the mean weight times the deformation. If yielding of the material occurs or scaling of the shell, a certain amount of energy is dissipated, equilibrium is destroyed, and new energy is developed by downward motion of the load thru increased deformation until a new condition of stability of equilibrium in established.

Scaling and cracking of the shell means a loss of potential energy stored and correspondingly larger deformations which are inadmissible in the practical structure. The addition of vertical steel prevents this checking and scaling and dissipation of energy because it provides a storage system of energy which is stable and in which the storage of energy by indirect stress can be depended upon.

Coaction between the hoops and the vertical steel reduces the deformation and hence the quantity of potential energy stored for a given load and correspondingly increases the efficiency of the structure as a load carrying mechanism.

9. Comparison of Test Data. Having pointed out in detail, the difference in action between these two kinds of columns, it is in order to compare test data. Column Table V, gives the reinforcement, percentage of steel, and test results by Withey* of a series of vertically reinforced and hooped columns with vertical steel flush with the ends and resting against bearing plates. These may be compared with the tests by Bach, and the tests at Phoenixville.

Taking up the comparison, first, with the series by Bach, the following point of difference is noticeable. The test results obtained by Withey can be substantially accounted for by considering the influence of the concrete and vertical steel alone. The test results by Bach, cannot be accounted for in this manner. Both series show that hooping adds toughness. Both indicate that larger vertical rods have more effect in raising the yield point than rods of too small diameter, which offer little resistance to lateral bending. The Bach tests, where the spiral pitch of is not too great, and where larger vertical rods are used with close spacing of spirals, are in good accordance with the Considère formula; while the tests by Withey are not in agreement but are in more close agreement with a different formula, which considers the manner in which the load is brought upon the steel, namely, by direct pressure instead of by bond shear as in usual operation of the column in practical building.

Comparing the tests by Withey with those at Phoenixville, the longitudinal reinforcement of the columns, in specimens, *J*, *N*, *P*, *O*, *R*, and *Q*, is substantially the same; while the hooping in the Phoenixville tests was much greater. The increase in ultimate strength in the Phoenixville test, for one percent of total steel, ranges from two to three times as much as in the tests by Withey. This

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*Bulletin of the University of Wisconsin, No. 466.

COL. TABLE V

Withey's Tests of Hooped Columns Longitudinal Steel Full Length of Shaft

Col. No.	Reinforcement				Percent Reinforcement		Core area sq. in.	Age in days	Mix	Ultimate strength lbs/sq.in P/A	Stress at yield pt. in core lbs/sq.in P1/A	P1/P	Compressive strength of cylinders lbs/sq.in
	Round Vertical Rods		Spiral		Longitudinal	Lateral							
	No.	Size	Size	Pitch									
1	2	3	4	5	6	7	8	9	10	11	12	13	14
C 1	0	0	¼″ rd.	1″	0	2.00	78.5	62	1-2-4	4,660	2,720	0.58	2,280
C 2	0	0	¼″ “	1″	0	2.00	78.5	62	1-2-4	4,390	2,330	0.53	2,190
C 3	0	0	¼″ “	1″	0	2.00	78.5	64	1-2-4	3,660	1,950	0.53	2,180
C 4	0	0	¼″ “	1″	0	2.00	78.5	65	1-2-4	3,410	2,330	0.68	2,150
D 1	9	⅝″	¼″ “	1″	3.50	2.00	78.5	58	1-2-4	4,470	3,290	0.74	2,150
D 2	9	⅝″	¼″ “	1″	3.50	2.00	78.5	60	1-2-4	4,200	3,480	0.83	2,130
D 3	9	⅝″	¼″ “	1″	3.50	2.00	78.5	61	1-2-4	4,970	3,860	0.77	2,380
D 4	9	⅝″	¼″ “	1″	3.50	2.00	78.5	62	1-2-4	5,360	3,670	0.68	2,350
H 1	0	0	No. 7	2″	0	0.50	78.5	57	1-2-3.5	2,330	1,950	0.84	2,040
H 2	0	0	No. 7	2″	0	0.50	78.5	57	1-2-3.5	2,140	1,760	0.82	*1,460
G 1	8	½″	No. 7	2″	2.00	0.50	78.5	49	1-2-3.5	3,320	2,710	0.82	2,100
G 2	8	½″	No. 7	2″	2.00	0.50	78.5	58	1-2-3.5	3,280	2,710	0.83	*1,420
I 1	8	11/16″	No. 7	2″	3.78	0.50	78.5	57	1-2-3.5	4,240	3,660	0.87	2,240
I 2	8	11/16′	No. 7	2″	3.78	0.50	78.5	57	1-2-3.5	4,080	3,280	0.80	2,120
J 1	8	⅞″	No. 7	2″	6.11	0.50	78.5	58	1-2-3.5	5,190	4,240	0.82	2,110
J 2	8	⅞″	No. 7	2″	6.11	0.50	78.5	58	1-2-3.5	5,050	4,240	0.84	2,000
L 1	0	0	No. 7	2″	0	0.50	78.5	57	1-2-3.5	2,680	1,370	0.51	1,780
L 2	0	0	No. 7	2″	0	0.50	78.5	58	1-2-3.5	2,600	1,370	0.53	1,760
K 1	8	½″	No 7	1″	2.00	1.00	78.5	57	1-2-3.5	4,050	2,710	0.67	2,100
K 2	8	½″	No. 7	1″	2.00	1.00	78.5	57	1-2-3.5	3,760	2,520	0.67	1,890
N 1	8	11/16″	No. 7	1″	3.78	1.00	78.5	57	1-2-3.5	4,050	3,470	0.86	1,880
N 2	8	11/16″	No. 7	1″	3.78	1.00	78.5	58	1-2-3.5	4,340	3,280	0.76	1,720
M1	8	⅞″	No. 7	1″	6.11	1.00	78.5	57	1-2-3.5	4,790	3,860	0.81	1,660
M2	8	⅞″	No. 7	1″	6.11	1.00	78.5	58	1-2-3.5	4,580	3,660	0.80	1,710
P 1	8	1″	No. 7	1″	8.00	1.00	78.5	57	1-2-4	6,760	5,560	0.82	2,380
P 2	8	1″	No. 7	1″	8.00	1.00	78.5	60	1-2-4	7,090	5,760	0.81	2,350
O 1	8	⅞″	¼″ rd.	1″	6.11	1.96	78.5	57	1-2-4	6,510	4,240	0.65	2,270
O 2	8	⅞″	¼″ “	1″	6.11	1.96	78.5	57	1-2-4	6,650	4,620	0.70	2,690
R 1	8	1″	¼″ “	1″	8.00	1.96	78.5	53	1-2-4	7,250	5,000	0.69	2,310
R 2	8	1″	¼″ “	1″	8.00	1.96	78.5	53	1-2-4	6,680	5,380	0.80	2,470
Q 1	8	1⅛″	¼″ “	1″	10.12	1.96	78.5	53	1-2-4	6,190	5,190	0.84	2,280
Q 2	8	1⅛″	¼″ “	1″	10.12	1.96	78.5	53	1-2-4	7,990	6,340	0.79	2,330
W1	0	0	0	0	0	0	86.6	52	1-2-4	2,660			2,600
W2	0	0	0	0	0	0	86.6	52	1-2-4	2,660			2,400
W3	0	0	0	0	0	0	86.6	51	1-2-4	2,480			2,250

Note: All spirals 10″ in diameter. Cols. C1-D4 are 120″-lg. All other cols. 102″-lg.

wide discrepancy, apparently can be accounted for only as above outlined, since the discrepancy is far too great for it to be possible to account for it on the ground of difference in the strength of the concrete. The concrete in the test of the Phoenixville specimens is undoubtedly closely comparable to the concrete of the specimens tested by Bach, and it is undoubtedly a somewhat better grade than the concrete in the specimens tested by Withey.

The conclusions that the authors draw from these tests are:

That it is bad practice to splice longitudinal reinforcing bars by bearing of one bar upon the other.

That it is better to lap bars of consecutive columns at the floor line in order that a natural adjustment may take place between the materials as the load is brought upon the concrete during erection.

That a column constructed by lapping the bars, so that the natural relations between the reinforcement and the concrete are conserved, that is, that indirect stress of the bond shear may be effective, substantially doubles the efficiency of the vertical steel; and hence this detail of reinforcing should be used. If this efficiency is to be counted upon, the diameter of the vertical bars should, for like reason, be limited preferably to $1\frac{1}{8}$ inch diameter, or in special cases of heavy work to $1\frac{1}{4}$ inches to $1\frac{3}{8}$ inch round bars.

That the hooping should not be spaced further apart than four inches for light pressures, and closer spacing should be used for heavy pressures.

That the spacing of the vertical bars should not exceed nine inches between centers, but if spaced further apart their efficiency should be considered to be reduced by fifty percent of the relative increase in spacing.

10. Formula for Columns, where the Load is Brought upon the Steel by Direct Bearing on Metal. Tests of this type of column show, up to the yield point, so-called, that the effect of the hooping is relatively small; that beyond the yield point the hooping is brought into play, giving the column a degree of toughness and ability to stand increased load without sudden failure, altho accompanied by considerable deformation. Neglecting the circumferential reinforcement and the concrete outside of it,

Let A = area of core occupied by concrete and longitudinal reinforcement having a steel ratio of p, so that

$A_s = pA$ = cross section of longitudinal steel.

$A_c = (1 - p)\,A$ = cross section of concrete core.

W = total load to yield point that column would carry without circumferential steel.

$e_1 = .00125''$ = average deformation per unit of length at yield point of column.

$W = (A_s\,E_s + A_c\,E_c)e_1$. Let $E_s/E_c = n$, then

$w = W/A = E_s\,e_1\,[\,(1 - p)/n + p]$

is the load at yield point per unit of cross section of concrete.

Let $E_s = 30{,}000{,}000$, and $n = 15$; then

$w = 37{,}500\,[\,(1 - p)/15 + p'\,] = 2500 + 35{,}000\,p$

The point here designated on the diagram of unit deformations and load is at intersection P of the tangents to the two slopes of the curve, Fig. 130.

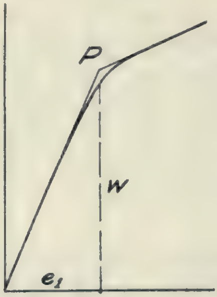

Fig. 130. Diagram Showing Elastic Curve of Column

Equation (1) is in good agreement with the series of tests made by Professor M. O. Withey, C. E., in 1910, at the University of Wisconsin,* with longitudinal reinforcement from 1 percent to 10 percent and circumferential reinforcement from 0.5 percent to 2 percent, given in Col. Table V. Professor Withey has written empirical formulas for several different percentages of circumferential reinforcement which differ slightly from this and from each other; but they do not refer precisely to the point P as above defined, but to a slightly smaller value of W. The point P does not actually fall on the test curve of unit deformations and loads, but is so related to it geometrically as to make its use advisable. The actual deformation at this load will be somewhat in excess of the assumed value of e_1.

On page 49, in Bulletin 466, University of Wisconsin, the following conclusions are drawn from the results of these tests regarding the behavior of columns with rigid metallic bearing for the longitudinal reinforcement built of concrete similar to those tested by Withey.

> 1. "Closely spaced spiral reinforcement will greatly increase the toughness and will considerably increase the ultimate strength of a concrete column, but will not materially affect the yield point. The ultimate strength under dead load will doubtless be somewhat less than the value obtained from a testing machine. Since spiral reinforcement should be employed principally as a factor of safety against sudden collapse, more than one percent does not appear to be necessary. On account of lack of stiffness in columns made from this grade of concrete and reinforced with spirals only, it seems necessary to use some longitudinal steel.
>
> 2. "Longitudinal steel in combination with such spiral reinforcement raises the yield point and ultimate strength of a column and increases its stiffness. As was shown in Bulletin No. 300 columns reinforced with longitudinal steel only are brittle and fail suddenly when the yield point of the steel is reached; but they are considerably stronger than plain concrete columns made from the same grade of concrete.
>
> 3. "From behavior under test of the columns reinforced with spirals and vertical steel and the results computed, it would seem that a static load equal to 30 to 40 percent of the yield point would be a safe working load."

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*Bulletin No. 466.

These tests are conclusive as regards the deportment of the column without vertical steel. Spiral reinforcement gives a large increase in the ultimate strength and while providing safety against sudden collapse, does not increase the yield point at which the fireproofing commences to scale, and hence does not permit material increase in the working stress over that of the vertically reinforced column types with ordinary ties.

11. Working Stresses. (*a*) *The case of no direct metallic bearings.* In the vertically reinforced and hooped column, in which the hooping follows closely the percentage of the vertical steel recommended previously, namely, .35 to .45 of the volume of the verticals, and the verticals are not less than three quarters of an inch in diameter, and spaced approximately as recommended, the yield point of the column may be taken as eighty percent of the ultimate strength. This would give a yield point value, for a column to which the Considère formula is applicable of .80 x 1.5 = 1.2 times the crushing strength of the concrete times the core area, plus eighty percent of the yield point value of the steel in verticals and hoops.

Since a column of this character may be depended upon with certainty, if ordinary care is used in erecting the work, the factor of safety of 2.5 may be employed. This would give, in round numbers, 800 pounds per square inch for the concrete of the core, 12,000 pounds per inch on the verticals, and 16,000 pounds per square inch on the hooping, treated as imaginary verticals, for the working stress, the hooping being assumed to be a drawn wire, and possessing a higher yield point than the material of the verticals.

With a 1 : 1½ : 3 mix, twenty percent higher values may be assigned to the core.

(*b*) *Metallic bearings.* Where vertical reinforcement is used, and the load is brought on the vertical steel by bearing on metal, there is much less certainty of uniformity of joint action between the matrix and the reinforcement than where the longitudinal bars are lapped; and hence a somewhat higher margin of safety should be allowed. Taking this margin of safety as the reciprocal of 3½, the writers would recommend a working stress not exceeding 700 pounds per square inch on the concrete plus 10,000 pounds per square inch on the vertical steel for this type of column with rigid end bearing, limiting the percentage of hooping to not less than one half of one percent nor less than twenty percent of the vertical steel, and the vertical steel limited to not more than eight percent of the column area. This factor of safety is based on the yield point value of the column.

12. Structural Columns Filled with Concrete. As steel is more rigid than concrete, the higher the percentage of the steel the less the assistance we may expect the concrete to be to the steel, and while the formulas given heretofore for the column in which the vertical steel is loaded in large part by direct bearing of metal on metal agrees with tests closely, it should be noted that the percentage of steel in these specimens was not high. Accordingly, it would seem conservative to reduce the allowance on concrete in cases where the percentage of steel exceeds eight percent of the combined area of the concrete and steel in proportion to the increase in steel above this percentage.

13. Concrete Columns Compared to Structural Steel. From the Phoenixville tests, it would seem that there is a higher degree of uniformity in the tested strength of reinforced concrete columns made with ordinary care and with well designed reinforcement than with average structural steel columns. The reason is that in the concrete column, we have a solid core. The larger the column the greater the strength, in strong contrast with some structural steel columns. In the structural steel columns, we have the uncertainty due to the form and make-up of the section. In the make-up of most small struts and columns constructed of steel and iron, we have a specific ratio of the area of the web and flanges consisting of channels, or in other forms, a complete circular or box section, such as the Phoenix column or the box made up of two channels and two plates. The semi-empirical formula for struts has been worked out for the radius of gyration of those forms of sections which are comparable in the case of laced channel struts to a certain distribution of metal between the web and the flange and a certain ratio of flange width to depth, which ratios, under standard sections, are comparable to the box and Phoenix sections. When there is a wide variation from these proportions, the formula based on the radius of gyration is inapplicable as is proved in the original design of the columns of the Quebec Bridge. In this case the area of the flange as compared to the web was about one tenth of this standard proportion between the area of the web and the channel, and there is no experimental data in existence covering the action of the latticed bars and secondary stress for such a combination.

In concrete work such uncertainties are eliminated by the uniform solid core. With ordinary care there can be no doubt about securing a solid casting, if the type of reinforcement selected is that recommended in this chapter, namely, one in which there is no obstruction whatever in the core to prevent the concrete from flowing freely and filling the same completely and so securing a solid casting. Improper handling of the concrete might lead to considerable reduction of the strength of the section, but the damage which might occur thru bad workmanship is if anything much less in concrete work than in steel construction and the confidence of the builder in the integrity of his concrete work should be correspondingly greater.

14. Wall Columns and Interior Columns in Skeleton Construction. Wall columns differ from the interior column in that the load comes to them from one side instead of equally or approximately equally from all directions as it does under a uniform load on the floors.

Whether the floor is beam and girder construction or flat slab construction supported directly upon columns in view of the monolithic connection between the floor and the columns certain eccentric loads or bending stresses are brought upon the wall columns to a greater extent than is the case with interior columns.

The amount of bending induced in the column will depend, first on the rigidity of the floor, and second, upon whether the resistance of the floor to deformation is furnished by beam action or circumferential slab action. If the resistance is by circumferential slab action, the effect upon the column is far less for the same deflection than it would be in case of beam action, because the slab tends to twist from all directions and so in a large measure the effect of the load in producing bending and deformation of the column is counter balanced by the loads coming to the column from the sides instead of being augmented by these loads so much as is the case in the beam action of one-way reinforcement.

Favoring Conditions. The walls of a building are made vertical and as the number of stories increases, the columns are reduced in diameter, but kept flush or vertical on the outer face. This produces an eccentric application of the column load from the upper stories upon the columns of the lower stories which in some large measure off-sets and holds in equilibrium the bending moment of the floor loads.

Little attention is usually paid to this difference in conditions. The wall column is commonly made approximately the same as the interior column except in the upper stories where it is possible to carry the wall columns up for the last three stories without reduction in size, making thereby a provision for the columns of these upper stories which would be most affected by the eccentric bending moment referred to.

Good practice would limit the dimensions of columns to sixteen inches as a minimum size in office buildings and in warehouse construction, eighteen inches as a minimum.

Provision of material in the beam to take the entire bending resistance figured as simply supported at the wall column and continuous over the interior columns will in no wise eliminate this bending stress in the column itself, and does not excuse failure to make such provision.

In fact if the beams are treated as beams fixed at the inner ends and free at the outer end, and the five rod type of reinforcement used in the Turner beam is adopted, the positive moment provided, for near mid span is $W L/14.4$ nearly, or $.07W L$, while the maximum positive moment in the beam freely supported and continuous over a number of spans in the end span requires a coefficient of from 0.07 for two spans to .08 for three spans, and .077 or .78 for a greater number, so that in this type of design there is nearly sufficient provision without considering the stiffness of the wall column. Nevertheless, in the view of the authors the provision recommended should be followed in practice.

Bending from Wind Pressure. In view of the monolithic character of reinforced concrete construction, wind has much less effect on a structure of this kind than is the case with the steel frame, since each floor forms a monolith of enormous lateral rigidity causing columns all to act in perfect unison. Where the building is narrow the columns should be fully spliced at the floor level which doubles the resistance of the column to flexure. Where the buildings are broad and only six to eight stories in height, provision of this character is unnecessary except in wall columns which should be well spliced.

15. Temperature Effect on Columns. Changes in temperature cause expansion and contraction of the concrete floor. This is taken up by bending in the column and the in and out motion of the walls where they are above the ground line. There is, however, a tendency for the basement walls to crack every thirty or forty feet because of the restraint of the surrounding earth. There is a further tendency for brick walls to crack where there is expansion if brick piers are used in place of well reinforced concrete piers in a long building. In a building over 300 feet in length, no dependence should be placed upon either a plain brick or a plain concrete pier without reinforcement to withstand temperature stresses of expansion and contraction of the floors since unsightly cracks and checks will very likely occur from this cause. In a building over 300 feet in length either a full, well reinforced concrete skeleton should be provided or if bearing walls are used the building should be cut at 250 foot intervals so that it will allow expansion and contraction without damage to the brick work or undue strain on the concrete of the floors and columns.

16. Economic Column Design. With reinforcing steel in the column figured at 2½ cents a pound and concrete at $6.00 per cubic yard, we may estimate the proportion of steel and concrete needed to obtain the most economic support from the standpoint of cost. Since a cubic foot of concrete weights 150 pounds and a cubic foot of steel 485 pounds, a cube of concrete weighing one pound would have a volume 3.2 times as great as a cube of steel of the same weight, and have a face nearly one and one-half times as great. On this basis, for equal cost in carrying load, $f_s/f_c = 37$, but in a fairly rich concrete $n = 10$ to 15, so that a load can be carried with less cost on a concrete pier than upon one of reinforced concrete, but in carrying the load in this manner, the diameter of the columns are greater, which is objectionable in that they occupy valuable room. Unbalanced bids are sometimes received on this basis. One puts in a bid using a column of a small diameter as specified, thus using more steel, but saving space; another makes a column larger by two or three inches and uses less steel and puts in a lower price. The owner is frequently persuaded to accept the large columns without giving the regular bidder an opportunity to revise his bid on the same basis.

It should be noted that the efficiency of the hooping varies as the diameter of the core, while the core area varies as the square of the diameter; hence a small increase in the diameter of the core permits a large decrease in the amount of hooping and vertical steel. Hooping costing 1.6 times as much as the vertical bars in a column in place has an efficiency 2.4 times that of the same cost of metal in vertical steel. On the other hand, certain relative proportions between the amount of the vertical steel and hooping are necessary to secure the best results, and these proportions should be adhered to if the formulas recommended are to be applied.

17. Column Formulas Dependent on Polar and Transverse Moments of Inertia. What the correct formula should be for compression members or columns of structural steel has for years been a subject of theoretical discussion and controversy. The formulas that have been adopted have, without exception, undertaken to determine what the working stress should be from the moments of inertia of the section that opposes flexural deformation, whereas most tests of structural steel columns clearly disclose failure by combined torsion and bending. Therefore, an accurate formula for a structural steel compression member must make its strength depend on polar as well as transverse moments of inertia. But the polar moment of inertia in many forms of steel columns reaches a very small value and the members are weaker than any computation which takes into consideration only their resistance to flexure in various directions and disregards their lack of stiffness under torsional deformation, indicates.

Because in the concrete column the section is solid and the polar moment of inertia is relatively so great that failure actually occurs by crushing and shear or flexure, the combination of polar moment with transverse moment of inertia has in this case no appreciable effect to diminish the resistance of the column. Accordingly, since plain crushing and transverse bending is by far the simpler phenomenon the uncertainty as to the safety of reinforced concrete columns when calculated by scientific formulas is far less than that involved in the use of the usual formulas to calculate the ultimate strength of structural steel members as commonly built.

18. Conclusion. In the preceding discussion of reinforced columns the authors have advocated higher working values for columns when vertically reinforced and hooped in proper proportions than is permitted by many building code regulations. On the other hand, nearly all such regulations allow working stresses upon the vertically reinforced column with ties higher than the authors consider safe in practice. The resistance of the hooping depends on the lateral expansion of the column under longitudinal compression.

In the column tests by F. P. McKibben and A. S. Merrill, Proc. Am. Conc. Inst., 1916, pp. 200-240, careful measurements were made to determine the lateral expansion of the column but as was the case in the beam tests by the Bureau of Standards no effort was made to ascertain the initial state of stress in the column produced during the chemical action of hardening.

Cracking or checking of the fireproofing occurs at low unit stresses where hooping only is used but with proper proportions of vertical steel the fireproofing remains intact under high intensities of load.

Because measurements under initial loading of columns show small hoop tension under compression not exceeding working stresses, the argument is advanced that the hooping provided to resist lateral expansion cannot be counted upon for elastic resistance of high stresses. The initial state of stress, however, is modified by repeated loading and continued use just as certainly as in the case of beams in which under initial loading the measured steel stress accounts for but a small part of the load moment. From this latter fact no one would conclude that the steel should not be counted upon to resist the load moment. Neither should similar data lead to the conclusion that hooping is not to be counted upon to resist high stresses in the column. Column reinforcement as well as beam reinforcement is largely affected by the state of stress induced during the hardening of the concrete, a matter yet to be fully investigated to complete our understanding of the co-operation of hooping and vertical reinforcement.

A thorough comparison of the working stresses allowed by different building codes throughout the United States by P. P. Furber, was published in Proc. Am. Conc. Inst., 1916, pp. 181-199 from which the following quotation with which the authors are in full accord is made.

> "Supposedly it is well known to structural engineers that columns vertically reinforced and hooped with sufficient spiral reinforcement of close pitch are much to be preferred to columns with vertical rods only, even in case ties are used, as they were not in the Edison buildings. In spite of this knowledge, columns without hooping are being built extensively throughout the country, although it is common practice to use ties or stays of small diameter rods or wire for lateral reinforcement, these being spaced about a foot apart as a rule. The reason for this inferior construction is found in the lower cost of that type as compared to the hooped column, when designed according to most building ordinances.
>
> Many building ordinances place a premium on this dangerous construction by allowing higher relative stresses on the concrete of such columns than on spirally reinforced columns, when we consider the increase in ultimate strength of the latter. Some have gone so far as to permit as great unit pressure on the concrete of a column without hooping as on a well designed hooped column. In other words, the concrete of tied columns is stressed beyond what should be permissible values, and the hooped column, instead of being given the benefit of its increased toughness and reliability, is actually put at a disadvantage because the allowed stresses are lower than tests have shown to be safe.
>
> The results of such building laws are obvious. Designers will use the cheaper type, except in some cases where it may be possible to control the design of the work with regard to actual strength secured in the structure rather than with a view to meeting the requirements of some building department. It may be said that a conscientious engineer will always work on the former principle, but this is not so, and will hardly be possible where there is an

> ordinance limitation. The engineer has no opportunity to use his best judgment. The department officials and framers of the codes have exercised that function so that all one can do is to follow the rules and get the cheapest thing possible.
>
> The discriminating engineer would rather be responsible for the construction of a spiral column—even with unit stresses considerably above those allowed by many building ordinances—than for columns without hooping, under the usual values for unit pressure. The difference between intelligent and inferior design may be judged better by the type of members and details selected than by the figured unit stresses.
>
> Most of the failures of concrete structures have occurred in buildings designed so that the figured unit stresses were not too high when judged by ordinance requirements, and the failures have been due to failures of the type of member rather than of the material. A large number can be blamed on the use of columns without hooping. On the other hand, the writer at least has never heard of the failure of a hooped column in a building. The continued service of hooped columns under pressures, which, according to some building laws, would be considered dangerous, leads one to believe that something is wrong with the building laws, particularly when reliable column tests indicate that the stresses allowed are conservative.
>
> Some may argue that the fact that a building stands up is no sign that it was well designed. This is quite true, and applies with force to many buildings in which rodded columns have been used. In considering designs in the true sense, that is, with reference to the type, it should be the duty of the engineer to avoid members that are subject to sudden failure."

The development of the stress analaysis of the solid column shows that as failure is approached the lateral swelling or bulging of the concrete at mid length of the column is a minimum; that it is a maximum at or in the vicinity of the quarter point of the length, and this is the reason that the research fellows of the experimental laboratories of our universities have erroneously concluded that the function of the hooping in developing strength of the concrete column is a small factor and the limitation of the amount which increases the efficiency with the vertical steel to one percent or such a matter is the outcome of this mistaken idea.

Stress analysis of the solid column shows that the closeness of the spacing of the hooping should be greatest at the end thirds of the column and may be reduced in the central portion of the column without loss of strength.

Because the reinforced concrete column is a solid section its computation is far simpler and more certain than that of the built up steel column and the results arrived at do not involve the inherent uncertainties of laminated plates more or less indifferently fastened together and treated by assumption as a solid together with the buckling and crippling tendency of common structural forms increased by the secondary stresses of lattice bars and the pantograph action of the lacing.

CHAPTER XXII

EYE BARS, PINS AND RIVETED JOINTS

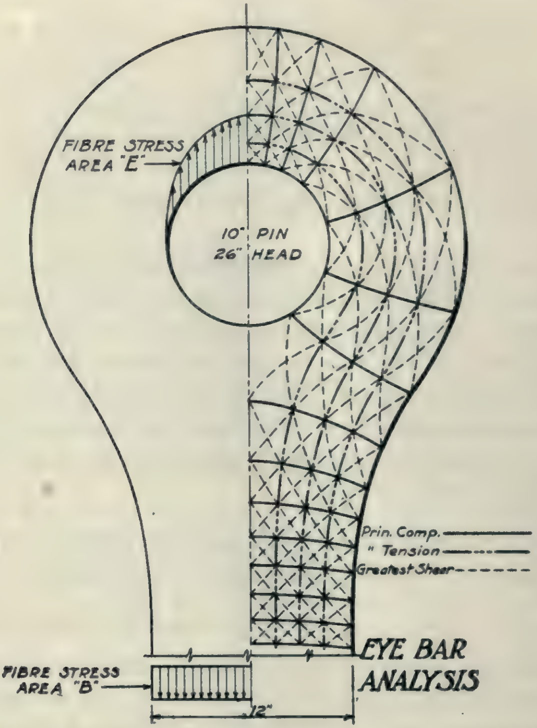
1. **Stress Analysis of Eye-Bar Heads** should disclose the reason for that change in form which accompanied the change from wrought iron to steel in their manufacture, the amount of excess section through the pin required over that of the body of the bar, the minmum permissible ratio of the diameter of pin to width of bar and the extent of inelastic stretch or permanent elongation of the piece end to end of holes under ordinary working stress for the given clearance between pin and the hole.

The accompanying figure shows such analysis for a 12 inch bar, a ten inch pin and 26 inch head, arrived at as follows:

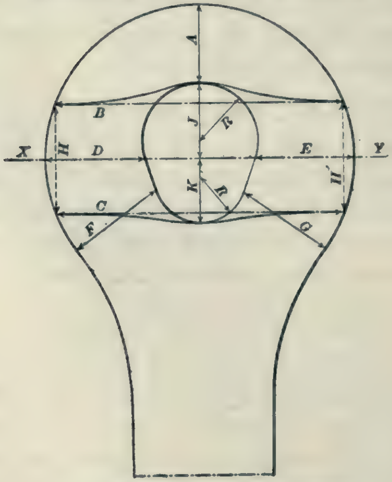
By Newton's third law, the fiber stress area B equals the fiber stress area E and we may assume that in the central body of the bar the stress is distributed with substantial uniformity permitting the uniform spacing of the coordinated principal tensions at B, the half bar being divided into four equal parts in the figure. The intensity of the average tensile stress across the section through the center of the pin would be the pull on the bar divided by the section so that the average tension through the head is less than that in the body of the bar by the amount of excess section.

The fiber stress area E is drawn on the basis that the bar has been stressed up to such a point that the metal has become distorted so that the pin' bears with approximate uniformity over its semi-diameter and for this condition we may divide the stress area E into equal parts locating the coordinate principal compressions at their point of intersection to the periphery of the pin.

Because the pressure on the pin tends to split or spread the material beyond the pin the curve of greatest shear from the side of the pin will extend to the center of the end of the head and the direction of the curves of greatest shear and their interesections with the principal tensions and compressions are obvious from this fact and a diagram of the principal stresses may be constructed as shown in the figure, with approximate accuracy.



·DEFORMATION OF HEADS OF EYE-BARS UNDER RUPTURE.



X-Y is line through center of original pin-hole

FIG. 3.

As wrought iron in tension is strongest with the grain and weakest across the grain, and more resistant to shear across the grain than with the grain this splitting tendency required an elongated head in forged wrought iron bars guaranteed to fail in the body and not in the head. The greater homogeneity of forged steel bars permits the adoption of the circular head in place of the elongated head of the old wrought iron member.

The effect of increased diameter of the hole over that of the pin for erection clearance reduces the bearing area to less than the pin diameter and the maximum unit bearing pressure increases in a greater ratio, so that before half the yield point value of the metal is developed in the body of the bar it is obvious that yielding must occur to increase the breadth of bearing on the pin unless the thickness of the head is of a much higher order than that of the body of the bar instead of the standard practice of making them substantially alike.

That such an elementary relation obvious from the necessary static balance of fiber stress areas E and B was regarded as *startling* Trans. Am. Soc. LXI, page 431 in a discussion of eye bars and as elucidated by the opinion page 448 that "the mathematical theory of elasticity applied to a diagram of strains in a solid leads to nothing practical" except * * * "mental satisfaction in forming a general conception of as to the manner in which forces are transferred in such structures" discloses that lack of comprehension of scientific means of investigation which tends to shackle the profession to rule of thumb, empiricism, and non-progressiveness.

The writer's deduction from stress analysis is that for a pin diameter three-quarters the width of the bar and for heads equaling the thickness of the bar the excess section across the pin should be approximately one and two-thirds that of the body of the bar.

When the diameter of the pin is increased to that of the bar this excess may be reduced to one and seven-sixteenths that of the body. When the pin diameter is one and one-third that of the width of the bar the excess may be reduced to one and three-tenths that of the body.

These relations desirable from the standpoint of avoidance of excess deformation of the head in testing to distruction have been followed closely in smaller sized bars which have been most frequently tested and have been skimped to some extent in the larger sizes more difficult to forge. This deficiency may be made good by increased thickness with the same dies but with more liberal upset allowance.

In computing deflections of the bridge truss it is conservative to consider the length of bar center to center the figured length plus twice the pin clearance.

Side by side with the stress line analysis of the eye bar head is reproduced Mr. Cooper's diagram of the head distortion at failure of the body of the bar.

Problem: A good exercise for the student is to construct an approximate chart of the curves of displacement by bisecting the angle between the greatest shears and major principal stress and compare them with the deductions from a rubber model. (See p. 441 Trans. Am. Soc. LXVI for such map under excess distortion) observing the rotational change in direction by excess deformation.

2. Secondary Stresses in Eye Bars arising from truss distortion have in general been disregarded by the practical engineer while erroneously over-estimated by theoretical writers.

Friction between the pin and hole preventing turning of the head on the pin through that slight angle necessary for the bar to adjust itself to the elastic deformation of the truss has been figured by Grimm, page 76 "Secondary Stresses in Bridge Structures", 1908. for the case of a maximum pin in a standard head as forty-five percent of P/A. This computation is arrived at upon the assumption of a coefficient of friction of .2 for the finished surfaces of the hole and bar with the customary coat of white lead and tallow whereas under test it is found to be about one-tenth that amount. See report of MacKay and Brown Vol. XXXII Trans. Eng. Inst. of Canada 1919. Moreover, in the equations by which the stress intensity is figured there appears a confusion of the moment of inertia for a pair of triangular fiber stress area of opposite sign in the width of a bar with that suitable to the case in hand, i. e., the moment of inertia of a trapezoidal area of one sign only. This confusion lead to the natural evolution of an imaginary secondary stress many times larger than disclosed to date in practical service or by experimental measurement.

3. The Maximum Strength of an Eye Bar in a Truss is less than the ultimate strength determined in the testing machine for the reason that as soon as the metal commences to yield a material percentage of its length the triangulation of the truss is so modified that failure must result if the bar be a main member.

In the case of counters in which the stress is more or less momentary the yield point of the bar may be occasionally exceeded and the member actually stretched a small amount without more immediately serious result than the loss of rigidity until the slack is taken up.

An interesting series of tests of carbon steel eye bars was made for the Board of Engineers for the Quebec Bridge. The test specimens of unannealed metal from these bars disclosed an ultimate strength of 65,000 pounds, a yield point slightly over 40,000 pounds, elongation of 23 percent in 8 inches and a reduction of area of 40 percent. The eye bars tested approximately 60,000 pounds ultimate strength, elastic limit from 21,000 to 24,000 pounds, a yield point varying from 25,000 to 28,000 pounds per square inch. In other words, the ultimate strength of the eye bar in the truss frame is but little in excess of 25,000 to 28,000 although it will not break under 60,000 pounds in the testing machine. As soon as the bar starts to yield if the load remains on the bridge the bar must not only carry the stress which causes it to yield but must resist also the shock of the dead weight and the live load descending through the height which the stretch permits the deflection to increase and this added to the stress exceeding the yield point may be regarded in general as sufficient to cause failure when taken into consideration with the distortion of other members which must follow a considerable elongation of the metal of the tie.

The appendix to the Report of the Board of Engineers of the Quebec Bridge, Col. 1, page 220, presents a painstaking investigation of the stresses in the bars by a 20-inch Howard gage, by Johnson extensometers and a Martin's mirror from which the following conclusions were drawn:

That the elastic limit and yield point occur at lower axial loads near the end than near the center of the bars. That considerable deviation from the linear relation of stress to strain takes place locally in the body of the bar at loads far less than those indicated as the yield point by the drop of the beam in a rapid continuous test. That counter poising the bars for their dead weight or loading them an additional amount equal to their weight resulted in no apparent change in the yield point and elastic limit or lowering of the ultimate strength.

As is apparent from the stress analysis of the head the axial resistance divides itself in passing beyond the pin and in so doing a state of shearing stress is induced in the shank of the bar toward the ends somewhat higher toward the pin than at the center of length giving rise to a curved deformation diagram when the measurements were taken on gage points about two feet from the center of the pin and practically straight deformation diagrams to nearly the yield point value when the extensometers were attached to the middle of the bar.

4. Combined Tension and Bending may be figured in a manner analogous to the inverted arch.

Let M equal the moment $WL/8$ due to the weight
" P " the pull on the bar
" d′ " the deflection when the pull equals zero
" D " the deflection under the pull P
" f′ " the fiber stress when the pull equals zero
" F " the increase of fiber stress over P/A under the pull P
" V " the middle ordinate of the curve of equilibrium of the centroid of tension in the bar under the pull P.

Then as in a string suspension $PV=M$ or $V=M/P$. But V is made up of the deflection D plus the displacement of the centroid of tension below the center of width of the bar, or D must be less than V and by Hooke's law $F : f' : : D : d'$. By the principle of rigidities

$$D=Vd'/(V+d') \text{ and } F=f\,D/d'$$

Example: For a 2″×10″ bar 40′0, stressed to 15,000 lbs. per square inch, $P=300{,}000$; $V=.544''$; $d=.785''$; $f'=4890\#$ per sq. in. and $F=2030\#$ per square inch, and

$$P/A+F=17{,}030 \text{ lbs. per sq. in.}$$

Now if P doubles in value $V=.272''$ and F reduces to 1280# per sq. in. $P/A+F=31{,}280$. i. e., the bending stresses decrease as the axial stresses increase; hence do not demand that careful consideration necessary in column action.

If the bar were turned flatwise instead of edgewise and subject to the same pull P, V would remain unchanged but D would approach V in value and F would be reduced nearly 80%. Such reduction in fiber stress under quiescent load would be achieved at a sacrifice of rigidity under moving load requiring an increase in impact allowance that would necessitate greater cross sectional area.

5. The Size of Pin is customarily made the maximum that the minimum sized head of the main eye bars of the truss permits, thereby economizing in thickness of pin plates on posts and chords while securing a more rigid joint without increase in weight of eye bar heads. When this is done the diameter is ample for shear and computation for bending only is requisite.

6. **Bending Moment on a Pin** arises from the fact that eye bars or members packed upon it cannot pull in line even though they act in the same plane. The bending moment is computed as in beam action, treating the center of action of the applied forces as applied at the center of the bearing area of the member upon the pin when the bearing is relatively narrow.

In general truss members connecting on a pin do not act in the same plane in which case the forces are resolved into their components on planes at right angles selecting one plane as the plane of the member carrying the greatest stress. Having figured the bending moment in these two respective planes they are to be combined by the parallelogram law, Chapter 1, Section 1, Art. 6.

The maximum moment is thus the square root of the sum of the squares of the maximum moments which occur at the same point along the pin in the two respective planes.

The moment on the pin is most readily computed by taking the extreme force times its distance to the center of the adjacent bar pulling in the opposite direction. Then the difference of stress between the second bar and the preceding constituting the shear times the next distance, and so on. The members should be packed so that the resulting moment is a minimum.

Because the diameter of the pin which constitutes the depth of the cantilever beam we are figuring upon is relatively great with respect to its length the shearing forces are distributed along the pin partly by shear and partly by bending. Moreover, the assumption that the pressure acts at the center of the bearing area results in an over estimate of the moment magnitudes. For these reasons it is customary to allow a fiber stress in bending upon the pin fifty percent greater than on the extreme fibers of rolled shapes, built sections, girders, and the like.

Intermediate grade metal of 80,000 pounds ultimate strength is to be preferred to structural grade for pins in order that the deformation or stretch of the distance from back to back of the holes will be a minimum. Where the pins are over 7 inches in disameter they are customarily forged and are drilled along their axis to insure soundness. This permits the attachment of a cap held in place by a through bolt instead of turning the pin down and threading for the usual recessed nut.

7. The Rivet is the simplest fastening for the transmission of stress between plates or shapes by shear. Rivets are formed from round rods at a red heat by an upsetting machine which cuts the heated rod to the desired length and upsets the finished cup shaped head at one end. When used they are heated to a red heat, entered in a hole through the plates to be connected and the second head is formed by hand or machine. In hand riveting the tail of the rivet is upset to fill the hole by hammering while the original head is bucked or firmly held on the opposite side. The head is finally set up and finished by a cup shaped die called a snap.

In the field of structural work the pneumatic hammer has almost entirely replaced hand riveting so that if properly heated there is little difference between the shop and field rivet. On the other hand in the field there is less uniformity of the heating and unless the rivets are driven at the proper temperature they may not be tight. A tight rivet or one which has cooled from 800 to 900 degrees Fahr. shrinks sufficiently to develop 15,000 to 18,000 pounds per square inch tension gripping the parts together if they have been well bolted in the first place so that a shoulder is not formed between the parts in driving the rivets. This shrinkage grip of the rivet develops friction between the parts connected and constitutes an element of strength of considerable magnitude when the rivet is first driven and may continue to exert part of this pressure provided loads are quiescent and the joint is not subjected to vibratory or reverse stresses in service. Quoting Carnegie Handbook, 1900, or like substance in 1923:

> "In transmitting stresses by means of rivets, it is customary to disregard the friction between the parts joined, as too uncertain an element to be relied upon to any extent. The rivets must then be proportioned for the entire stress which is to be transmitted from one plate or group of plates to the other, and they must be of sufficient size and number to present ample resistance to shearing and afford sufficient bearing area so as not to cause a crushing of the metal at the rivet holes. This latter condition, while generally observed for pins, is very often entirely overlooked in riveted work. Its observance, in most cases of riveted girders with single webs, determines the size and nunber of rivets to be used and frequently makes it necessary to adopt a greater thickness of web than would otherwise be required. Thus, if the web is 5/16" thick, the rivets connecting the same with the flange angles have a bearing value of only 3,520 lbs. for a 3/4" rivet, while their shearing value is $=2\times 3{,}310=6{,}620$ lbs. per rivet, the rivets being in double shear. Consequently while the usual thickness of web of floor beams for railway bridges is 3/8", it sometimes becomes necessary for shallow floor beams to increase the thickness to 1/2" and even 5/8", in order that the pressure of the rivets upon the semi-intrados of the rivet holes be not excessive between the points of support of floor beam and of application of the load (in which space the transmission of strain from web to flanges takes place).

But the sound theory presented in the foregoing quotation is not consistent with the "standard connections" illustrated in Fig. following. Shop rivets in these beam connections are figured for the value of 30,000 pounds per inch in enclosed bearing and 12,000 pounds for shear with bearing on one side at 24,000 lbs. per sq. in.

The standard connection angles were changed in the 1913 edition of Carnegie from those of the 1900 edition reducing the shop rivets for 15" web connections from 5 to 4, for the 12" connection from 5 to 3, for the 18" and 20" from the five rivets to four.

If the theory that the frictional grip is to be disregarded and the bearing and shearing value of the rivet is to be depended upon solely then for standard material having a strength of 55,000 pounds per square inch and a guaranteed minimum yield point of 27,500 pounds the use of a working stress in bearing of 30,000 lbs. per sq. in. is ten percent greater than the elastic limit or yield point of the material and the margin of safety for elastic strength has passed through zero and changed sign.

8. A Review of the Rising Bearing and Shear Values for Rivets used during the past thirty years is of interest.

When steel commenced to replace wrought iron in 1890 customary values for shear on rivets in railroad bridges was 6,000 pounds per square inch and for bearing 12,000 pounds and the values reduced 25 to 30 percent for field connections. As steel came into use this was raised to 7,500 for shear and 15,000 for bearing while in building work 9,000 in shear and 18,000 in bearing seems to have been the highest value thought of or considered within reason.

In 1906 the Cooper specification for railroad bridges allowed 9,000 pounds per inch for shear and 15,000 pounds per square inch for bearing on truss members reducing this amount 20 percent for the floor system.

The specification of 1908 Department of Railways and Canals of Canada permitted 10,000 pounds in shear and 20,000 pounds in bearing but the material was to be medium steel.

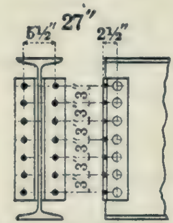
Pencoyd Handbook of 1898 gives but three tables for rivets, shear at 6,000, bearing at 12,000; and shear at 7,500 bearing at 15,000; and single shear at 11,000 and bearing at 22,000, the higher value used in connection with an impact allowance of eighty or ninety percent in bridge floor systems of medium steel.

STANDARD CONNECTIONS

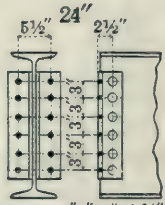
from

Carnegie Pocketbook of 1913 to '23

BEAM CONNECTIONS



2 Angles 4″×4″×½″× 1′-8½″



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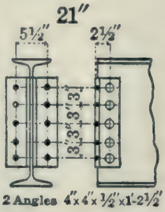

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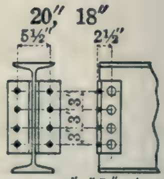
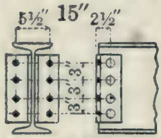
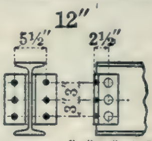


2 Angles 4″×4″×7/16″×0′-11½″

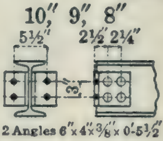
Rivets and bolts-¾″diam.



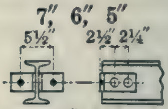
2 Angles 4″×4″×7/16″× 0′-11½″



2 Angles 4″×4″×7/16″× 0′-8½″



2 Angles 6″×4″×⅜″× 0′-5½″



2 Angles 6″×4″×⅜″× 0′-3″

LIMITING VALUES OF BEAM CONNECTIONS

I Beams		Value of Web Connection	Values of Outstanding Legs of Connection Angles					
			Field Rivets			Field Bolts		
Depth, Inches	Weight Lbs. per Foot	Shop Rivets in Enclosed Bearing, Pounds	¾″ Rivets or Turned Bolts, Single Shear, Pounds	Min. Allowable Span in Feet, Uniform Load	t, In.	¾″ Rough Bolts, Single Shear, Pounds	Min. Allowable Span in Feet, Uniform Load	t, In.
27	83	66800	61900	18.4	⅝	49500	23.1	⅝
24	80	67500	53000	17.5	⅝	42400	21.9	⅝
	69½	52700	53000	16.3	⅝	42400	20.2	⅝
21	57½	40200	44200	15.5	9/16	35300	17.6	⅝
20	65	45000	35300	17.6	⅝	28300	22.1	⅝
18	55	41400	35300	13.3	⅝	28300	16.7	⅝
	46	29000	35300	15.0	½	28300	15.4	⅝
15	42	36900	35300	8.9	⅝	28300	11.1	⅝
	36	26000	35300	11.1	7/16	28300	11.1	9/16
12	31½	23600	26500	8.1	9/16	21200	9.0	⅝
	27½	17200	26500	10 3	7/16	21200	10.3	½
10	25	27900	17700	7.4	⅝	14100	9.2	⅝
	22	20900	17700	6.9	⅝	14100	8.6	⅝
9	21	26100	17700	5.7	⅝	14100	7.1	⅝
8	18	24300	17700	4.3	⅝	14100	5.4	⅝
	17½	18900	17700	4.4	⅝	14100	5.5	⅝
7	15	11300	8800	6.2	⅝	7100	7.8	⅝
6	12¼	10400	8800	4.4	⅝	7100	5.5	⅝
5	9¾	9500	8800	2.9	⅝	7100	3.6	⅝

ALLOWABLE UNIT STRESS IN POUNDS PER SQUARE INCH

Single Shear	Rivets . . . . Shop 12000		Bearing	Rivets—enclosed . . . . Shop 30000
	Rivets and Turned Bolts. . . Field 10000			Rivets—one side . . . . Shop 24000
	Rough Bolts . . . . Field 8000			Rivets and Turned Bolts, Field 20000
				Rough Bolts . . . . Field 16000

t=Web thickness, in bearing, to develop max. allowable reactions, when beams frame opposite.

Connections are figured for bearing and shear (no moment considered).

The above values agree with tests made on beams under ordinary conditions of use.

Where web is enclosed between connection angles (enclosed bearing), values are greater because of the increased efficiency due to friction and grip.

Special connections shall be used when any of the limiting conditions given above are exceeded—such as end reaction from loaded beam being greater than value of connection; shorter span with beam fully loaded; or a less thickness of web when maximum allowable reactions are used.

Today these values have increased from the 6,000 pounds for shear and 12,000 pounds in bearing for iron rivets of 1890 to 12,000 or 13,500 for shear on steel rivets as soft as the iron rivets of 1890 while the bearing values have been increased from 12,000 pounds for wrought iron possessing greater compressive resistance than ordinary present day steel to 30,000 pounds per square inch. Obviously if these values keep on increasing something is liable to happen.

Higher values of working stresses are legitimate provided a steel of greater resistance is specified which warrants the corresponding increase in stress.

9. The Resultant Principal Fiber Stress on a Rivet limiting or determining its elastic working capacity is to be found by combining its axial tension due to the shrinkage stress of hot driving, the bending effect of the offset of shearing forces combined with the shear stress acting upon it.

When the shearing force is small the resistance opposed to sliding is the frictional resistance equaling the tension on the rivet from shrinkage times the coefficient of static friction on the rough surface in the neighborhood of .35 to .45 percent which would indicate an initial slip or permanent set at 5,000 to 6,500 pounds apparent shear figured upon the rivet sections. When the joint suffers this initial slip the maximum fiber stress is the resultant of the initial tension; the bending and the shear strains.

If the magnitude of each of these kinds of stress be known they may be combined by the formulas applied to the shaft under torsional shear and bending, i. e., the fiber stress of bending is added to the initial axial tension and this state of stress at the extreme fiber in tension is to be combined with the state of shearing stress.

Now the bending moment on a rivet or pin equals the cube of the diameter times substantially one-tenth the fiber stress in bending. (Art. 15, Chapter I, Sec. I). Hence for $^3/_4$ inch rivets connecting $^1/_2$ inch plates under an assumed bending fiber stress per square inch which will produce this bending moment assuming the center of bearing one-third the thickness of the plate from the sliding surface ;

$M = 27/64 \times 0.1 \times 10{,}000 = 425$ lbs. or the shear $= 2{,}550$ on the three-quarter inch rivet equals 5,800 lbs. per sq. in. Maximum shear stress $S_s = \sqrt{11^2 + 5.8^2}$ in Kips equals 12,500 lbs. per square inch when the shirinkage tension at slip is taken at the value of 12,000 lbs. per square inch.

The maximum tension $St = 11 + \sqrt{11.^2 + 5.8^2} = 23{,}500$ lbs. per sq. in., i. e., the elastic tensile stress is nearly TWICE AS GREAT as the resultant elastic stress in shear and FOUR TIMES AS GREAT as the unit shear stress computed on the rivet cross section. It should be noted that in this computation 12,000 lbs. per square inch used as the shrinkage tension in the outer fiber of the rivet at slip is 8,000 lbs. per square inch less than Unwin's estimate of 20,000 lbs. per square inch average shrinkage stress against 15,000 to 18,000 lbs. per square inch by the writer and on the outer fiber at slip of 12,000 lbs. The computation for bending follows the usual method for pins but assumes the center of pressure at a less rigorous location than the center of the bearing. The formulas for combination of these component states of stress are accepted and taught in works on the mechanics of material and are given in Kent's Mechanical Engineer's Pocketbook.

With the strenuous effort now made to keep working stresses within the limits of elasticity of the material of which we construct great bridges even to the laborious determination of minor secondary stresses that exist more predominantly in the imaginative assumptions of the computer than in the finished structure such major stresses as are readily approximated in the rivet by the elastic theory should receive careful considerations.

If this be done the values which F. H. Lewis (Eng. Club. Phila. 1891. Kent Pocketbook 1903 p. 380) selected for shear and bearing in the light of the thorough investigations of the resistance of riveted joints during the age of wrought iron and the early introduction of steel will return to general use.

10. The Ultimate Shear Strength of a Rivet Rod in the ordinary testing machine runs substantially eight-tenths its tensile resistance. In the joint where by upsetting to fill the hole the area of the rivet is increased from one-sixth to one-eighth its ultimate strength averages about three-quarters of the tensile strength of the original section. As the yield point resistance of the outer fiber of the rivet is exceeded in tension the shrinkage grip of the rivet decreases and the shear strain assumes an increasing value until rupture occurs by detrusion.

11. The Maximum Stress Supported by Any Given Rivet relative to the stress on the sections riveted together depends on the uniformity of the distribution of that stress among the group of rivets which resist the axial stress on the parts.

If the group of rivets is symmetrically disposed with respect to the axis of the members connected and the stress on the members connected is axial and not bending then each rivet in a compact group may be treated as carrying its proportionate part of the stress or load. If this is not the case a proper increase in the number are needed to provide for the moment brought upon the group by the unsymmetrical configuration. Better if possible design or arrange the group in a symmetrical manner and avoid the labor of computation involved in checking the strength of old structure badly designed in this regard.

The inequality in distribution of the stress over the rivets through the angles of a stringer connection to the web of the floor beam in old railway bridges is brought forcibly to the attention of the engineer responsible for old overloaded bridges on several thousand miles of road. The splitting down of the connection angles half way from the top to the bottom in service under the bending of too shallow stringers has been repeatedly noticed and corrected by the use of more rivets with a wider gage so that elastic bending of the angle flanges relieve the intensity of the tension on the rivets.

In the light of such experience the assumption of perfectly uniform distribution of the shear upon the rivets of the Carnegie beam connections appears grotesque indeed, a dream of the arm chair engineer rather than the product of practical experience of the man on the job.

Where the group of rivets is not compact, a reduction of three percent in the permissible value for shear should be made for each five diameters in distance that the center of the splice exceeds twenty diameters of the rivet from the end.

12. Working Stresses in bearing on pins and rivets should permit the same margin of overload that the body of the member is designed to withstand. Bearing on large pins and enclosed bearing on rivets should not exceed 15,000 pounds per square inch for metal averaging 60,000 pounds ultimate strength nor 18,000 pounds per square inch for intermediate grade steel of 65,000 to 75,000 pounds ultimate resistance.

Mr. Howard's valuable paper, Trans. Am. Soc. C. E. Vol. LXXIII, pp. 442-443, discloses yielding of the pin plates under slightly less than 25,000 pounds per square inch bearing on the pin and obviously 24,000 lbs. per square inch bearing on the A. R. E. and M. W. Assoc. specification is too high leaving no margin between the working stress and the yield point value.

Bearing values of 12,000 pounds for soft steel and 15,000 lbs. for a one side bearing intermediate grade steel correspond to the foregoing limits because of the increased displacement of the rivet by bending.

Elastic Working Stresses of One Kind in Shear on soft steel rivets of ultimate strength over 46,000 pounds ultimate 56,000 pounds per square inch, cannot materially exceed 6,500 pounds per square inch. On medium steel 60,000 to 68,000 pounds ultimate strength, 7,500 pounds per square inch. On field rivets for railway bridges driven with pneumatic hammer 500 pounds per square inch less.

The present practice of using dead soft metal for rivets results in a lower initial slip and yeild point value of the joints. In 1896 the Pottsville Iron and Steel Company used medium steel for shop rivets and the tested values were from fifteen to twenty-five percent higher than with soft steel and no difficulty was found in satisfactorily driving them in the shop.

Tests comparing nickel and soft steel riveted specimens is presented in an appendix, Vol. I, Government Board of Engineers report on the Quebec Bridge. The greater strength of the joints with the nickel steel is due to the greater tenacity and hardness of the nickel alloy over the soft steel. Perhaps high resistance rivet steel for special work may economically be manufactured with a small amount of chrome to increase its hot ducility while securing greater hardness with medium low carbon.

Working Stresses for Alternating Tension and Compression should be reduced one-half where the minimum stress equals or exceeds half the maximum. The values may be increased in proportion as the minor stress decreases in proportion to the major from this limit.

Where the rivet length exceeds four diameters the strain upon the rivet in shear and bearing should be reduced by ten percent for each diameter increase in the length of the rivet.

Conclusion: The exaggerated shear and bearing values on rivets may be traced to a certain confusion of ideas relative to impact. The first high values for rivet shear in bearing appear to be those associated with Pencoyd specification which amounts to an increase in the figured shear of over 90 percent on a twenty-five foot stringer and 50 percent on a three hundred foot main truss chord but if this increase in the stress on the principal member is necessary to provide for impact upon what line of reasoning is to be presumed that the same impact allowance should not be applied to the rivet which makes good the value of the sectional area of the part?

Confusion and misunderstanding regarding the origin of excessive working values is such that stresses in excess of the elastic range of the metal have been adopted in buildings and structures subjected to quiescent loads. What will become of this frictional resistance of the joint on the enclosed bearing in case it is exposed to heat with even an insignificant fire in the building is a question which the purchaser of the building and the architect as well should be interested in and it seems incumbent on those to explain who have the temerity to recommend such abnormal values for the detail design of important building constructions.

Admittedly the stresses to which the rivet is subjected are complex. The tensile stress from shrinkage may readily be greater than that which we have figured. Again in upsetting the rivet in the hole through punched plates there are frequently offsets in the plate joints where the hole is not reamed out and the rivet in bending may act as a nicked section instead of one presenting a uniform and constant moment of inertia. All of these uncertainties warrant the conservative adoption of relatively low working stresses instead of endeavoring to strain the material as close to its yield point value as it can be expected to go and still have the frame stand up.

If the pin values were figured as the rivet has been computed for 12,000 lbs. shear and 24,000 lbs. bending the maximum fiber stress would equal $12+\sqrt{12^2+12^2}=28{,}964$ lbs. per sq. in. But for the pin to be comparable to the ordinary rivet the maximum 16 in. eye bars on a 12 in. pin would need to be about 10 inches thick instead of 2 in., the maximum made; i. e., the shear value in general is far smaller than specified and the length of the cantilever developing bending is of a much lower order for the length ratio L/D in the case of the pin.

The flexural stress developed in the pin is relatively less than in a beam of normal L/D ratio to which our flexure formula strictly applies. The different proportion of the rivet or relation of its diameter to the thickness of the plate connected renders the computed flexural stress approximately in accord with that actually developed.

In noting that the computation of the combined stress in the rivet and pin is the same as that of combined bending and shear in the shaft it is in order to point out the difference between beam action and the relation of bending and shear in rivet and pin resistance. In beam action the shear passes through zero where the bending moment passes through a maximum. Moreover, the greatest shear is at the neutral plane of the beam and is zero at the outer fiber because these shears are the product of predominant transverse stress in contradistinction to predominant shearing stress. With predominant shearing stress the maximum shear stress occurs at the locus of the cutting edge in place of the center of thickness, hence the shear stress and the bending stress pass through a maximum at approximately the same position on the extreme fiber as a diagram of the distribution of the principal stresses would pictorially disclose.

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Turner, Claude Allen Porter
Elasticity and strength
of materials in construction

Engineering

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